

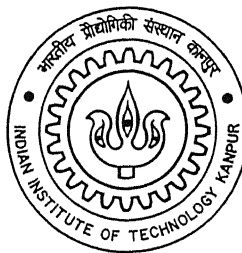
Uplift Capacity of Inclined Anchor Pile

**A Thesis Submitted in the Partial Fulfilment of the
Requirement for the Degree of**

MASTER OF TECHNOLOGY

By

PRADIPTA KUNDU



To the

**DEPARTMENT OF CIVIL ENGINEERING
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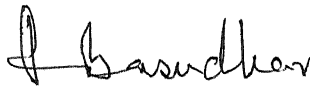
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CERTIFICATE

It is certified that the work presented in this thesis entitled “**Uplift Capacity of Inclined Anchor Pile**”, has been carried out by **Mr. Pradipta Kundu** (Roll No Y3103034) under our supervision and that this work has not been submitted elsewhere for a degree.



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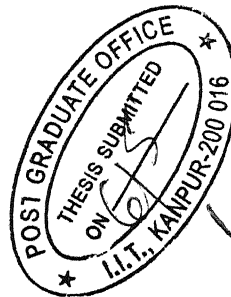

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LIST OF SYMBOLS

Following symbols are used throughout this thesis unless otherwise specified.

L	Length of the inclined pile
D	Diameter of the pile shaft
D_b	Diameter of the enlarged base or the anchor
β	Inclination of Pile with the vertical
E_s	Young's Modulus of the soil material
E_p	Young's Modulus of the pile material
ν_s	Poisson's Ratio of the soil
n	Number of pile elements of equal length
P	Uplift load
P_U	Ultimate Uplift Capacity
p_j	Stress at any element j on the pile shaft
p_b	Stress at the enlarged base or the anchor
$s\rho_{ij}$	displacement of the soil adjacent to the pile at i resulting from the stress p_j on an element j
I_{ij}	Displacement factor for element i due to stress at j
I_{ib}	Displacement factor for element i due to stress at base
I_{bj}	displacement factor for base element due to stress at j
I_{bb}	Displacement factor for base element due to stress at base
$s\rho_i$	Displacement at i due to all the stresses
$s\rho_b$	Displacement at base element
$\{s\rho\}$	Soil displacement factor
$\{p\}$	Pile stress vector
$[I_s]$	$(n+1)$ by $(n+1)$ matrix of soil displacement influence factors
$\{p\rho\}$	Pile displacement vector
λ	Axial displacement of the pile
$[I]$	Unit vector

I_t	Influence factor for tensile load
p_{\max}	Maximum shear stress on the pile shaft
$p_{b\max}$	Maximum shear stress on the base of the pile
K	Lateral earth pressure co-efficient
σ_v'	Average overburden pressure
δ	Angle of friction between pile and soil material
N_q	Bearing capacity factor
ϕ	Angle of shearing resistance of soil
D_R	Relative Density of the soil medium
τ_a	Limiting shear stress of soil
C_U	Uniformity coefficient
C_C	Coefficient of curvature

ABSTRACT

Large inclined tensile loads often act on the foundation of retaining walls, anchors for bulkheads, bridge abutments, piers and offshore structures. Inclined or battered piles with anchors are economically used in these foundations, wherever they are required to carry large inclined pulling forces. In this thesis, a study on the uplift capacity of incline anchor piles has been made. To predict the ultimate uplift capacity of the inclined anchor piles an analytical approach has been developed here based on theory of elasticity and Mindlin's solution of point load acting with in a semi-infinite elastic half space. The linear elastic theory has been modified for taking into account the pile soil slip along the interface and the ultimate uplift load has been determined. An experimental program has also been carried out to determine the uplift capacity of the inclined anchor piles. The solutions obtained from the proposed theory have been compared with the experimental results. Finally, parametric study on the ultimate pullout capacity of inclined anchor piles are presented based on the analytical solutions obtained for different pile inclinations, different anchor diameters, varying soil density and for rough and smooth piles.

CHAPTER 1

INTRODUCTION

1.1 General

The use of piles is one of the most common methods of overcoming the difficulties of founding on soft soils. Piles are columnar elements in a foundation which have the function of transferring load from the superstructure through weak compressible strata or through water, onto stiffer or more compact and less compressible soils or onto rocks. Pile foundations are generally used to support compressive loads from superstructures such as buildings, bridges etc. Structures like transmission towers, mooring systems for ocean surface or submerged platforms, tall chimneys, jetty structures and underground tanks transmit not only heavy compressive loads but they are also subjected to considerable amount of uplift forces. If a structure is tall or high and is subjected to large lateral forces such as wind or wave loads, the resulting moments could induce tension in some of the piles supporting the structures. These structures need footings which can anchor them with the competent strata. Under-reamed piles and anchor piles are being extensively used in such cases depending on the situ conditions. Large inclined loads can also act on the foundation of retaining walls, anchors for bulkheads, bridge abutments, piers and offshore structures. Inclined or battered piles with anchors can be economically used in these foundations, wherever they are required to carry large inclined pulling forces.

1.2 Scope of the Study

Uplift resistance of an inclined pile with base enlargement is a complicated phenomenon involving variables like length of the pile, pile inclination, shaft diameter of the base enlargement, pile friction angle, density of the foundation medium and the angle of shearing resistance of the soil. The ultimate uplift resistances of vertical and inclined piles are usually predicted by considering the shearing resistance mobilised along the rupture surface in addition to the weight of the soil bounded by rupture surface. But all the available methods are either underestimate or overestimate the ultimate uplift capacity of the piles. Elastic theory using Mindlin's Equations are successfully applied to determine the load carrying capacity of piles subjected to compressive load. The same can be used in case of piles subjected to tensile forces to determine the uplift capacity.

From an overview of the literature on the subject showed that lots of work on the uplift capacity of vertical piles had been conducted. Some works had also been carried out to analyse the behavior of battered pile subjected to axial uplift force. From those works, it can be concluded that with increasing the inclination of the pile with the vertical the ultimate uplift capacity decreases, in general conditions, unless the soil is overconsolidated or mechanically compacted. But in several field situations, inclined piles are subjected to large lateral forces which could induce tensile forces on the piles. In such situations use of inclined piles may become uneconomical because ultimate uplift capacities of these piles are less compared to those of vertical piles. To increase the uplift capacity of the inclined piles, anchor can be provided at the base of the pile. Till now only some works had been done on the ultimate uplift capacity of the vertical anchor piles, but not much work on the uplift capacity of the inclined anchor piles are available. Therefore, it is intended to undertake such a study in this

thesis. To determine the ultimate uplift capacity of the inclined anchor piles an analytical approach based on elastic theory using Mindlin's solution has been developed here.

1.3 Organization of the thesis

The motivation of taking up of the present study has been discussed in the above section. In this section the organization of the thesis is presented as follows indicating the scope of each subsequent chapter.

In chapter 2 the available literature on uplift capacity of piles are briefly presented.

In Chapter 3, a model to predict the uplift capacity of inclined anchor piles based on theory of elasticity and Mindlin's solution of point load acting with in a semi-infinite elastic half space are described.

Chapter 4 deals with the experimental set up and adopted procedure in measuring the uplift capacity of model anchor piles.

Chapter 5 pertains to the presentation of the results, computed (using the method developed) and measured (from the model experiments) and parametric studies on the uplift capacities of the inclined anchor piles are also presented.

In chapter 6, based on the above discussion conclusions are drawn with regard to the applicability of the method to predict correctly the uplift capacity of anchor piles.

Finally scope of future work is also outlined.

CHAPTER 2

LITERATURE REVIEW

2.1 General:

Analysis of piles may be carried out using methods based on limiting equilibrium, theory of elasticity and plasticity and finite elements. Most of the analysis that have been developed based on theory of elasticity/ plasticity and FEM are on compressive piles. The methods that have been developed to estimate the ultimate pile load capacity under compression may be used to predict the same but under tensile loads. As such, the general approaches that have been developed for compressive piles are also included in the review of literature and presented under different sections depending on the method of analysis.

2.2 Analysis Based on Elastic Theory Using Mindlin's Equation

Soils and rocks are not ideal elastic materials in that stress and strain are not linearly related, strains are not fully recoverable on reduction of stress, and strains are not independent of time. However, at least it can be said that strains in soil increase as stresses increase. Furthermore, the assumptions of anything more complicated than a linearly elastic material for the soil in the pile-soil continuum situation would generally lead to unduly complicated theory lacking useful generality. The use of linear elastic theory is therefore expedient and should be sufficiently accurate for engineering purposes, provided that elastic "constants" are employed that are appropriate to the particular problem.

The basic elastic response of the soil from which the solutions for elastic piles in elastic soils can be derived is given by Mindlin's set of equations for the stresses and displacements throughout an elastic half-space resulting from horizontal or vertical point load applied at a point beneath the surface.

Elastic based analysis had been employed by several investigators: for example, D'Appolonia and Romualdi (1963), Thurman and D'Appolonia (1965), Poulos and Davis (1968), Mattes and Poulos (1969), Butterfield and Banerjee (1971), Randolph and Wroth (1978). In most of these approaches, the pile is divided into a number of uniformly loaded elements, and a solution is obtained by imposing compatibility between the displacements of the pile and the adjacent soil for each element of the pile. The displacements of the pile are obtained by considering the compressibility of the pile under axial loading. The soil displacements are obtained in most cases by using Mindlin's equations for the displacements within a soil mass caused by loading within the mass.

The main difference between the various methods lies in the assumptions made regarding the distribution of the shear stress along the pile. D'Appolonia and Romualdi (1963), Thurman and D'Appolonia (1965) assume the shear stress at each element to be represented by a single point load acting on the axis at the centre of each element. Poulos and Davis (1968), Mattes and Poulos (1969), Butterfield and Banerjee (1971) consider the shear stress distributed uniformly around the circumference of the pile. The latter appears to be more satisfactory of the two methods mentioned, especially for shorter piles. However, for relatively slender piles, there is a very little difference between solutions based on the above two representations of the shear stress.

2.2.1 Piles Subjected to Compressive Loads

Poulos and Davis (1968) analysed the settlement behaviour of a single axially loaded incompressible cylindrical pile in an ideal soil mass using Mindlin's equations. By considering the pile as a number of uniformly loaded circular base, solutions were

obtained for the distribution of shear stress along the pile and the displacement of the pile. Influence factors were presented for the settlement of a pile in a semi-infinite mass and in a finite layer, and the effects of length to diameter ratio of the pile, Poisson's ratio of the soil and the depth of the soil were also examined. They also examined the effect of an enlarged base on the behaviour of a single pile and it was shown that the effect was of major significance only for relatively short piles. The elastic analysis was extended to include the effect of local shear failure between the pile and cohesive soil and the load settlement behaviour up to general failure was given for typical cases.

Mattes and Poulos (1969) extended the previous work for compressible piles. According to the authors, there were many cases in which the pile cannot be considered even approximately as incompressible in relation to the surrounding soil. Such cases might include timber, concrete, and steel-pipe piles in stiff clay. In these cases, the compressibility of the pile material itself may provide a significant portion of settlement of the pile and should be taken into account in analysing the behaviour of the pile.

In their work, linear elastic theory was employed to analyse the behaviour of a compressible floating pile of circular cross section in an ideal elastic soil mass. The vertical displacement of the soil due to the shear stress along the pile was obtained by double integration of the Mindlin's equation for vertical displacement around the surface of each pile element. In calculating the displacement of the pile itself, only pure axial compression of the pile is considered. The authors examined the influence of the compressibility of the pile on the load transfer along the pile and the settlement of the pile with particular reference to the difference in behaviour between a

compressible and an incompressible pile. They had shown that the influence of compressibility on the behaviour of the pile was likely to be significant for relatively slender piles but small for relatively short piles.

Butterfield and Banerjee (1971) analysed the response of rigid and compressible single piles embedded in a homogeneous isotropic linear elastic medium by a rigorous analysis based on Mindlin's solutions for a point load in the interior of an ideal elastic medium. The previous analysis of Poulos and Davis (1968) and Mattes and Poulos (1969) were based on the following assumptions, in addition to those of ideal elasticity:

- a) The pile shaft load was replaced by uniform vertical shear stress on the surface of each of a suitable number of small cylindrical pile elements.
- b) The pile base was assumed to be a smooth disc, not necessarily the same diameter as the shaft, across which the base load was uniformly distributed.
- c) The disturbance of the continuity of the elastic half space due to the presence of the piles was ignored.

The analysis presented by Butterfield and Banerjee (1971) was capable of eliminating assumption (b) and (c) and proved that assumption (b) had a significant effect on the response of underreamed piles and very short piles. They considered both the vertical and radial displacement compatibility for the settlement of the pile, whereas the previous work of Mattes and Poulos (1969) considered only the vertical displacement compatibility. They also extended their analysis to analyse axially loaded rigid and compressible pile groups with floating caps spaced in an arbitrary manner.

Poulos and Madhav (1971) analysed the axial displacement of a battered pile in ideal elastic soil mass by using the elastic theory. The analysis was considered in two stages:

- (i) A battered pile subjected to an axial load
- (ii) A battered pile subjected to a normal load and a moment.

For the first case, the axial displacement of the pile and the soil at the centre of each element were evaluated equated, and the resulting equation is being solved to obtain the unknown stresses and the displacements. In evaluating the soil displacements, the unknown force on each element was resolved into vertical and horizontal components, and vertical and horizontal displacements due to each of these components were calculated using Mindlin's equation. These displacements were then combined to give the axial displacement.

For the second case, the following conditions had been studied:

- (i) Pile free at top and tip and subjected to
 - a) Normal load, and
 - b) Moment
- (ii) Pile fixed at the top but free at the tip, and subjected to normal load.

It was found that the axial displacement of a battered pile subjected to axial load, and the normal displacement and rotation of a pile subjected to normal load and moment, were virtually independent of the batter of the pile, at least for the range of batter angles employed in practice. The method of analysing the single batter pile was also extended to the consideration of the group containing the batter piles.

Prajapati and Char (1977) extended the elastic approach employing Mindlin's equation to study the behavior of batter piles. They presented an analysis based on the

elastic approach for battered piles subjected to horizontal loads embedded in cohesionless soil having constant soil modulus and Poisson's ratio with the depth. They modified the equation, used by Spillars and Stoll (1964) to compute the soil displacements associated with vertical piles to get the displacements in case of single batter piles, subjected to horizontal load at the ground surface. Pile displacements had been obtained from the equation of flexure of a thin strip, expressed in finite difference form. The resulting equations were solved to calculate the normal deflections, bending moments and the soil reactions along the batter piles. Suitable modifications were also done to allow for three linear variation of soil modulus with depth. The theoretical solutions obtained were compared with the available experimental results on model batter piles. The solutions obtained for the deflection and the bending moment seemed to agree well with the experimental results in case of mild batters if constant soil modulus was assumed, though it overestimated the soil reactions. On the assumptions of hydrostatical variation of soil modulus with depth, the deflection and bending moment were overestimated while the soil reactions had realistic distribution along the pile.

2.2.2 Piles Subjected to Tensile Loads

Madhav and Poorooshab (1988) analysed the deformation behavior of single pile subjected to tensile load. The pile was assumed to be rigid and the soil as a linear elastic continuum. Due to the stresses at the pile-soil interface, the vertical displacement of soil at each node was calculated using the Mindlin's equation. Satisfying the compatibility of the displacements of soil and pile, the pile displacement δ was obtained as

$$\delta = -\frac{P}{E_s d} I_t \dots\dots\dots (2.1)$$

Where, P was the axial pull on pile, d the pile diameter, E_s the modulus of deformation of soil and I_t was the influence factor due to tensile load. I_t was evaluated as a function of the length to diameter (L/d) ratio and Poisson ratio of the soil. They had also conducted some tensile load tests to evaluate unit shaft resistance and the deformation modulus of the soil. The modulus of deformation computed from the results of the tensile load test compared well with those from other types of test like plate load or axial compression, and lateral load on a pile.

2.3 Vertical Piles Subjected To Axial Uplift Force

When a vertical uniform pile is subjected to axial uplift load in cohesive soil the ultimate uplift resistance $Q_u(\text{gross})$ is found by conventional method as,

$$Q_u (\text{gross}) = C_a A_s + W \dots\dots\dots (2.2)$$

Where, W is the weight of pile and C_a is average adhesion along pile shaft.

And

$$Q_u (\text{net}) = C_a A_s \dots\dots\dots (2.3)$$

Similarly the uplift resistance of straight-shafted pile in cohesionless soil is assumed to be dependent on the skin friction between pile shaft and the soil and net uplift capacity $Q_u (\text{net})$ of vertical curricular pile of diameter d, and embedded depth L, in sand, is expressed as,

$$Q_u (\text{net}) = P_{av} \cdot \pi d \cdot L = (0.5 K_s \tan \delta \gamma L) \pi d \cdot L \dots\dots\dots (2.4)$$

Where P_{av} = average skin friction = $(0.5 K_s \tan \delta \gamma L)$,

K_s = co-efficient of earth pressure,

δ = angle of pile Friction and

γ = effective unit weight of soil.

Various attempts have been made to evaluate the value of K_s in Eqn. (2.4). Several other analytical methods has been introduced to calculate the uplift capacity of vertical piles subjected to axial pull and number of model scale laboratory and field tests on piles were also conducted to study the behavior of those piles.

Meyerhof and Adams (1968) developed an approximate theory with some simplifying assumptions in respect of failure plane. The theory was derived for uplift capacity of strip or continuous footings on sand and was then modified for circular and rectangular footing. It was then extended for piles and pile groups also. In sand the experimental uplift capacity co-efficient increased with angle of internal friction of soil (ϕ) and ratio of depth to width of footings (D/B). However, the experimental uplift capacity coefficient remained constant for large values of D/B for circular footings. For long rectangular footings it is independent of ϕ .

Vesic (1970) suggested that limiting skin friction in pile is the same in tension and compression and beyond a critical depth of $10D$ in very loose sand and $20D$ in very dense sand, average skin friction (f_f) depends on the relative density (D_r) of the surrounding soil and the mode of placement of the pile.

For driven pile

$$f_f = 0.08 (10)^{1.5} D_r^4 \dots\dots\dots (2.5)$$

For bored pile and piers in dry sand,

$$f_f = 0.025(10)^{1.5} D_r^4 \dots\dots\dots (2.6)$$

Meyerhof (1973a) presented an analytical method to evaluate the uplift capacity of circular piles and this was an extension of earlier work of Meyerhof and Adams (1968) on uplift capacity of footings. He introduced an uplift coefficient K_u and uplift capacity was given by,

$$Q_u = \frac{1}{2} (K_u \tan \delta \gamma \pi d L^2) \dots\dots\dots (2.7)$$

For a particular value of angle of shearing resistance ϕ of the soil, the value of K_u was shown to increase with an increase in slenderness ratio, L/d , up to a maximum value and there after remains constant. The depth where the value of K_u attains maximum value was designated as the critical depth. Beyond this critical depth uplift capacity should be analysed by using the limiting uplift coefficient, implying a linear increase of average skin friction with a further increase in the depth of embedment. This limiting uplift coefficient however, had the increasing tendency with the increase of ϕ .

Das and Seeley (1975) presented some experimental results for the vertical uplift capacity of rough buried model piles in loose granular soil having relative density of 21% and angle of internal friction of 31° , they concluded that :

1. The variation of unit uplift skin-friction for piles was approximately linear with depth up to a critical embedment ratio, beyond which it reached a limiting value. For short rigid piles, the uplift capacity could be calculated by using Meyerhof's uplift coefficient.
2. The final 'skin friction' for tension piles in loose granular soils used for this test program was attained at a depth of about 10-12 times of pile diameter. However, this might not be true for all granular soil.
3. Ultimate skin friction for tension loading was 0.76 times that for compressive loading when evaluated by Vasic's empirical relationship.

Awad and Ayoub (1976) reported some uplift test results on model piles driven in loose sand. Uplift capacity was given as,

$$Q_u = \pi d L^2 \delta \gamma \mu_s \tan^2 (45^\circ + \phi / 2) \dots\dots\dots (2.8)$$

Where, $\mu_s = 0.33$ for cast in place piles and driven timber and concrete piles with rough surface. Using, a value of $\mu_s = 0.323$ corresponding to $\delta = 2/3(\phi)$, they claimed satisfactory correlation between theoretical and experimental results.

Chattopadhyay and Pise (1986a) proposed a theoretical method for predicting the uplift capacity of circular pile embedded in sand based on a few assumptions regarding its failure surface and curved failure surface through the soil as shown in Fig (2.1) has been proposed. Failure surfaces for different values of angle of friction (ϕ) and pile friction angle (δ) were evaluated. Net uplift capacity as well as average skin friction values was evaluated on the basis of different parameters like length-to-diameter ratio (λ), pile friction angle δ , angle of shearing resistance ϕ .

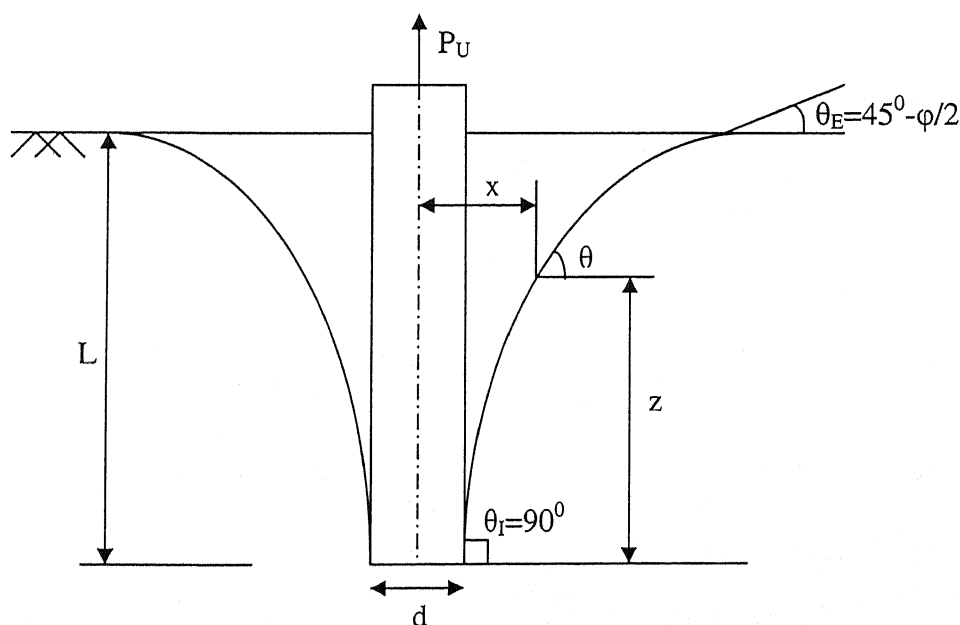


Fig 2.1 Pile and Failure Surface (Chattopadhyay and Pise, 1986a)

The gross ultimate uplift capacity is given as,

$$Q_u (\text{Gross}) = A \gamma \pi d L^2 \dots\dots\dots (2.9)$$

Where $A = f(X, L, Z, d, K, \theta, \phi)$, called gross uplift capacity factor.

Net uplift capacity is given by,

$$\begin{aligned} Q_u (\text{Net}) &= Q_u (\text{gross}) - \text{weight of pile} = A \gamma \pi d L^2 - (\pi d^2/4) \gamma L \\ &= \gamma \pi d L^2 (A - 1/4\lambda) \\ &= A_1 \gamma \pi d L^2 \dots\dots\dots (2.10) \end{aligned}$$

And average skin friction $(P_{av}) = A_1 \lambda L = A_1 \gamma \cdot \lambda d$

$$\text{Or } P_{av} / \gamma \cdot d = A_1 \lambda \dots\dots\dots (2.11)$$

In which A_1 = net uplift capacity factor and $K_S = 2A_1/\tan\delta$, where θ = inclination of failure surface. Reasonable agreement was observed between the experimental and theoretical results. Also field-tests results agreed with the predicated values of ultimate uplift resistance and skin failure. The proposed analysis was capable of predicating the nonlinear variation of dimensionless load factor with λ , whereas Meyerhof's analysis (Meyerhof 1973) predicted linear variation.

It has been found that the critical depth of embedment beyond which the average skin friction attains a constant value depends not only on the angle of shearing resistance of sand but significantly on pile friction angle.

Sharma and Pise (1994) proposed an analytical approach to estimate uplift capacity of anchor pile in sand under axial pulling loads. Uplift resistance of a pile with base enlargement was calculated with various variables like length of the pile, shaft diameter of the base enlargement, pile friction angle, density of the foundation medium and the angle of shearing resistance of the soil and considering the shearing resistance mobilized by rupture surface. For design of these piles various methods

were available. However, many of them either underestimate or overestimate the uplift capacity values. The authors therefore have done model test to study the behavior of anchor piles under axial pulling.

Based on the model test results authors observed that (i) the uplift capacity of the anchor pile increase with L , B/d and δ , (ii) δ had substantial influence upon the uplift capacity even if the pile had a base enlargement. However, the influence of δ decreases as B/d ratio increases. If B/d was small, uplift resistance from the pile surface friction had considerable magnitude. Therefore, in this method suggested by authors the net uplift capacity of anchor piles was calculated by summing up the resistance offered by the shaft of pile and resistance offered by the annular area of the base enlargement. They were estimated independently by using the analysis of Chattopadhyay and Pise (1986) for axial uplift capacity of vertical piles and horizontal anchors respectively. Considering these two uplift resistance together authors suggested net uplift capacity of the anchor pile as,

$$P_{nu} = A_1 \gamma \pi d L^2 + \gamma L N_{ql} A' \dots\dots\dots (2.12)$$

Where, A_1 was net uplift capacity factor depends on λ , ϕ and δ , γ was effective unit weight of the soil, d was the diameter of the pile, L was depth of embedment. N_{ql} breakout factor which was a function of relative depth, L/B and ϕ and A was the annular area of the base enlargement i.e. $A' = \pi/4 (B^2 - d^2)$, where B was diameter of the base of footing. In the above equation 1st term was the uplift resistance of a circular vertical pile embedded in sand and 2nd term was the ultimate uplift capacity of circular anchor plate embedded in sand.

It was observed that the test results were closer with the values of uplift capacity of anchor poles estimated by the method proposed by authors than those predicted by other methods. From the test results authors also concluded that smooth piles required

relatively higher displacement compared to the rough piles to mobilize the ultimate resistance and rough piles offer more resistance than smooth piles.

2.4 Inclined Piles Subjected To Axial Uplift Force

A small number of analytical studies have been made regarding batter piles subjected to axial pull.

Meyerhof (1973b) presented an analysis to determine the axial pull out resistance of batter piles. Extending the earlier theory of vertical uplift capacity of foundations (Meyerhof and Adams, 1968), Meyerhof reported experimental and theoretical study results on the axial pull out capacity of vertical and batter piles and anchors. Neglecting the self-weight of pile, the net pull out resistance, Q_u of a batter pile is expressed as:

$$Q_u = (C_a + \sigma'_0 K_u \tan \delta) A_s \dots\dots\dots (2.13)$$

Where, K_u = uplift coefficient.

σ'_0 = the average effective overburden stress = $\gamma D/2$

C_a = adhesion angle

δ = wall friction angle

A_s = pile surface area = $\pi.d.L$

d = diameter of pile

L = embedded length of pile

γ = effective unit weight of soil and

D = vertical depth of embedment.

According to Meyerhof for a given friction angle ϕ the values of K_u did not differ much for moderate pile inclinations ($\beta \leq 45^\circ$), but decrease more rapidly for greater inclinations.

According to **Tran-Vo-Nhiem (1971)** within above-mentioned limit the ratio of the uplift of an inclined pile of inclination β , to the uplift capacity of a vertical pile having equal vertical depth of embedment, neglecting the value of K_u was expressed as:

$$Q_{ui} / Q_{uo} = \sec \beta \dots\dots\dots (2.14)$$

Where Q_{ui} = axial uplift capacity of an inclined pile

And Q_{uo} = axial uplift capacity of a vertical pile.

Adams and Klym (1972) reported an axial pull out test on an aluminium pile of diameter 10 cm and length 65 cm embedded in densely packed silica sand, with batter angle of 35° . Experimental results indicated on appreciable change in the value of K_u for vertical and inclined pile. It was also observed that the axial pull out capacity for inclined piles decreases with an increase of the inclination angle.

Awad and Ayoub (1976) conducted experiments on batter piles. The results of tests indicate a decrease in axial pull out capacity with increasing pile inclination β . An empirical relation was suggested for the ratio of the net axial pull out capacity of a batter pile, Q_{ui} , with respect to that of a vertical pile, Q_{uo} as:

$$(Q_{ui} / Q_{uo}) = (\cos \beta / \cos \beta + \tan \beta) \dots\dots\dots (2.15)$$

Hanna and Afram (1986) investigated the pullout capacity of single rigid vertical and batter piles subjected to axial loading in sand on instrumented model piles. The

effect of pile inclination on the pull out capacity has been explained by means of variable mobilized passive earth pressure on the pile's perimeter. They had showed small variations but near constancy of the pull out capacity with batter angle upto 30° . The following empirical relation has been proposed.

$$(Q_{ui} / Q_{uo}) = \cos (\beta/2) \dots\dots\dots (2.16)$$

Chattopadhyay and Pise (1986b) proposed a generalized theory to evaluate the axial uplift capacity of a circular inclined pile embedded in sand. According to authors the ultimate uplift resistance of a circular batter pile embedded in sand, with an inclined angle of β to the vertical was,

$$Q_{ui} = K \tan \delta \gamma d L^2 I_i \dots\dots\dots (2.17)$$

$$\text{Where, } I_i = \int_0^{\pi/2} \left[\frac{\alpha^2 + \tan^2 \phi_1}{\sec^2 i + \tan^2 \phi_1} \right]^{1/2} d\phi_1 \dots\dots\dots (2.18)$$

K = Coefficient of lateral earth pressure

$$\alpha = \cos \beta + (\sin \beta / K)$$

$\tan \phi_1 = (\tan \phi / \cos \beta)$, ϕ is the angle of internal friction of sand.

The ratio of the ultimate uplift capacity of the inclined pile to that of a vertical pile for equal vertical depth of embedment was given as,

$$\alpha_D = \sec^2 \beta (2I_i / \pi) \dots\dots\dots (2.19)$$

Similarly ratio of ultimate uplift capacity of an inclined pile to the of a vertical pile with same length was,

$$\alpha_L = (2 I_{iv} / \pi) (K_{iv} / K) \dots\dots\dots (2.20)$$

Where, K_{iv} = the coefficient of earth pressure for vertical pile of length $L \cos \beta$.

Values of l_i / π and α_D were plotted for piles of equal vertical depth of embedment against inclination angle with different values of K and it was observed that with increase in inclination of pile α_D was increased, thus uplift capacity also increased.

To substantiate the theoretical results, axial uplift of circular piles of inclination varying between $2^\circ - 45^\circ$ in dense sand with 3 different pile surface characteristics and 3 different pile length having slenderness ratio $\lambda = L/d$ ranging from 11.44 to 39.18 were done. Inclined piles having equal vertical depth of embedment the theoretical values of uplift capacity suggested by authors showed better agreement to the experimental results than those predicted by Meyerhof's analysis (1973) But experimental uplift capacity of inclined pile of same length increased up to a certain inclination of pile and thereafter it decreased, a maximum value of uplift capacity occurs at inclination pile lies between $15^\circ - 22\frac{1}{2}^\circ$. The proposed theoretical value given by authors also followed qualitatively the experimental trend and it predicted optimum value of inclination at which uplift capacity of pile attained a maximum value. They also depended on pile friction angle and slenderness ratio. On the other hand Meyerhof's analysis predicted values which decreased with increase in inclination of a pile and values were independent of pile friction angle and slenderness ratio. Though estimated values of uplift capacities were less than the experimental values, but they were much closer than those predicted by Meyerhof's analysis.

Hanna and Madhav (2000) developed a new unified theory to predict the pull out capacity of single batter pile as compared to that of the vertical pile. Initial stress condition existing into granular soils for driving pile, an-isotropic undrained strength and variation with depth for cohesive soil was considered during the study. The net

axial pull out capacity of inclined piles with an inclined angle β with vertical could be expressed as,

$$Q_{u\alpha} = \pi l \left(\gamma' \frac{L^2}{2} \right) \left\{ f (\sin^2 \alpha + K_0 \cos^2 \alpha) \tan \partial - (1 - K_0) \sin \alpha \cos \alpha \right\} \dots (2.21)$$

Where, f = a multiplication factor for the densification effect,

K_0 = Coefficient of lateral earth pressure at rest of the soil. According to Mayne and Kulhaway (1982); Hanna and Ghaly (1992) for normally consolidated soil, $K_0 = (1 - \sin \phi)$ and for over consolidated soil $K_0 = (1 - \sin \phi) \text{OCR}^{\sin \phi}$.

If $\beta = 0$, the net uplift capacity, Q_{u0} of a vertical pile is obtained as,

$$Q_{u0} = \pi l \left(\gamma' \frac{L^2}{2} \right) K_0 \cdot f \cdot \tan \partial \dots (2.22)$$

Where $K_0 f = K_u$, the lateral uplift coefficient proposed by Meyerhof (1973). Now the ratio of uplift capacity of inclined pile to the uplift capacity of vertical pile expressed as,

$$\mu_u = \frac{\left\{ f (\sin^2 \alpha + K_0 \cos^2 \alpha) \cdot \tan \partial - (1 - K_0) \sin \alpha \cos \alpha \right\} \cos \alpha}{f K_0 \tan \partial} \dots (2.23)$$

Thus the pull out capacity of an inclined pile could be calculated by knowing pull out capacity for a vertical pile and value of r .

The authors had carried out a parametric study and they concluded that,

1. For low K_0 values (K_0 : 0.5 – 1.0) the axial pull out capacity decreased uniformly with increasing pile inclination.
2. The axial pull out capacity increased slightly for $\beta < 22.5^\circ$ and decreased for $\beta > 22.5^\circ$ for medium dense or lightly cover-consolidated soil. (K_0 : 1.0 – 1.25).
3. The axial pull out capacity increased for $\beta < 25^\circ$ in case of very dense or highly over consolidated soil (K_0 : 1.25 – 2.0).

2.5 Summary

From the above discussions on the brief literature review related to the present work, it can be concluded that lots of work had been done on the analysis of vertical piles under tensile loads. Some works had also been carried out to analyse the behavior of battered pile subjected to axial uplift force. These are summarized here in a tabular form.

Table 2.1 Summary of the Literature Review

AUTHORS	APPROACH
Poulos and Davis (1968)	Analysed the settlement behaviour of a single axially loaded incompressible cylindrical pile in an ideal soil mass using Mindlin's equations.
Meyerhof and Adams (1968)	Derived an approximate theory for uplift capacity of strip or continuous footings on sand and then modified for circular and rectangular footing. It was further extended for piles and pile groups.
Mattes and Poulos (1969)	Analysed the behavior of compressible as well as incompressible floating piles of circular cross section using linear elastic theory in an ideal elastic soil mass.
Butterfield and Banerjee (1971)	Analysed the response of rigid and compressible single piles embedded in a homogeneous isotropic linear elastic medium by a rigorous analysis based on Mindlin's solutions. They considered both the vertical and radial displacement compatibility for the settlement of a pile.
Poulos and Madhav (1971)	Analysed the axial displacement of a battered pile in an elastic soil mass subject to axial loading, and of the normal displacement due to the normal loading and moment. They found that these displacements were almost independent of the angle of batter of the pile over a practical range of batter angle.

AUTHORS	APPROACH
Tran-Vo-Nhiem (1971)	Expressed the ratio of the uplift capacity of an inclined pile with an inclination of β to the uplift capacity of a vertical pile having equal vertical depth of embedment as $\sec\beta$.
Adams and Klym (1972)	Concluded that the axial pull out capacity for inclined piles decreased with an increase of the inclination angle based on some experimental results on model inclined piles.
Meyerhof (1973a)	Presented an analytical method to evaluate the uplift capacity of circular piles. He introduced an uplift coefficient K_u and uplift capacity was given by, $Q_u = \frac{1}{2} (K_u \tan\delta \gamma \pi d L^2)$
Meyerhof (1973b)	Extended his previous theory of vertical uplift capacity of foundations to inclined anchors and piles under axial loads. The behaviour of inclined tension piles was analyzed by the use of the limiting uplift coefficients.
Das and Seeley (1975)	Presented some experimental results for the vertical uplift capacity of rough buried model piles in loose granular soil.
Awad and Ayoub (1976)	Experimentally studied the ultimate uplift capacity of vertical and inclined piles by model tests.
Prajapati and Char (1977)	Presented an analysis based on the elastic approach for battered piles subjected to horizontal loads embedded in cohesionless soil having constant soil modulus and Poisson's ratio with the depth.
Chattopadhyay and Pise (1986a)	Proposed a theoretical method for predicting the uplift capacity of circular vertical pile embedded in sand based on a few assumptions regarding its curved failure surface.
Hanna and Afram (1986)	Showed small variations but near constancy of the pull out capacity with batter angle up to 30° . The following empirical relation has been proposed. $(Q_{ui} / Q_{uo}) = \cos(\beta/2)$

AUTHORS	APPROACH
Chattopadhyay and Pise (1986b)	Proposed a generalized theory to evaluate the axial uplift capacity of a circular inclined pile embedded in sand. They had done some model tests and the estimated values from the analytical approach had been compared with the field and model test results.
Madhav and Poorooshab (1988)	Presented a method of analysis based on elastic theory (using Mindlin's equation) of a pile subjected to a tensile load. The pile was treated as rigid and the soil as an elastic half space.
Sharma and Pise (1994)	Proposed an analytical approach to estimate uplift capacity of vertical anchor pile in sand under axial pulling loads. The net uplift capacity of anchor piles was calculated by summing up the resistance offered by the shaft of pile and resistance offered by the annular area of the base enlargement.
Hanna and Madhav (2000)	Developed a new unified theory to predict the pullout capacity of single batter piles as compared to that of the vertical one. The analysis accounted for the initial stress condition existing in soil prior to driving the pile into granular soils and anisotropic undrained strength and its variation with depth for cohesive soils. For the values of $K_0 < 1$, the pullout capacity was decreased with increasing inclination angle of a batter pile while it was increased for $K_0 > 1$.

From an overview of the literature on the subject showed that even though lot of work on the uplift capacity of vertical piles as well as inclined piles have been conducted not much of work on the uplift capacity of inclined piles with anchors are available. Therefore, it is intended to undertake such a study in this thesis.

CHAPTER 3

METHOD OF ANALYSIS FOR FINDING THE UPLIFT CAPACITY OF SINGLE INCLINED ANCHOR PILE

3.1 General

In Chapters 1 and 2 the importance of estimating the uplift capacity of individual piles has been highlighted detailing the scope of the present thesis and a review of the literature on the subject have been respectively been presented. Based on the literature review it was concluded that application of Mindlin's solution for a point load acting with in a semi-infinite elastic half space can be conveniently extended to determine the uplift capacity of individual piles. Such an approach is very rational and theoretically sound and has some edge over conventional approaches as using this method one can develop the complete load-deflection diagram and finally determine the ultimate uplift load capacity from such a diagram. In addition one can estimate also the stresses and displacements over the entire region of concern. The importance of determination of stresses and displacements from engineering point of view is very important and needs no emphasis. In this chapter after introducing the problem, the general analysis procedure to estimate the ultimate uplift capacity of individual piles is presented as follows.

3.2 Statement of the Problem

An embedded anchor pile is considered to be a cylinder, of length L , shaft diameter d , and base diameter D_b , inclined with vertical by an angle β and loaded with an axial uplift force P at the ground surface (Fig 3.1). The soil is assumed to be homogeneous, isotropic, and an elastic half space having elastic parameters E_s and ν_s . These parameters are assumed unaltered by the presence of the pile. For this stated problem, the pile top displacement and the load distribution along the pile shaft are measured.

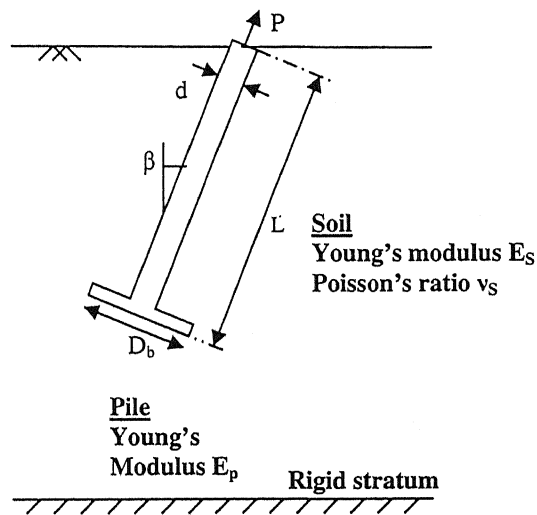


Fig 3.1 Definition sketch

3.3 Assumptions

The following assumptions are made in analyzing an inclined anchor pile:

- (i) The pile shaft load is replaced by uniform shear stress on the surface of a suitable number of small cylindrical pile elements.
- (ii) The soil is initially considered to be an ideal homogeneous isotropic half-space, having elastic parameters E_s and ν_s , that are not influenced by the presence of the pile.

- (iii) The pile and soil are initially stress-free, and that no residual stresses exist in the pile resulting from its installation.

Holloway et al. (1975) emphasized the importance of residual pile soil stresses on pile behavior and on the interpretation of pile load tests, and suggested a method for evaluating such stresses. However, in order to reduce the complexity of the analysis here, the assumption of an initially stress free pile is adopted.

- (iv) Throughout the analysis only the compatibility of axial displacements is considered.

Ideally consideration should be given to compatibility of both vertical and radial displacements. However, this more complicated analysis gives solutions that are generally almost identical with those from a simpler analysis that considers only vertical displacement compatibility.

- (v) At the pile-soil interface it is assumed that no slip occurs, that means the movements of the pile and the adjacent soil must be equal.

3.4 Method of Analysis

Mindlin (1936) presented solutions for both the vertical and horizontal displacements caused due to a vertical point load and a horizontal point load respectively within the interior of a semi-infinite, elastic, isotropic, and homogeneous soil mass. These solutions are the corner stones of the elastic continuum approach and these are used here for the analysis of load settlement behavior of axially loaded inclined pile. General solution procedure to find the stresses and displacements for axially loaded piles is available in standard reference book (Poulos and Davis, 1980). The same solution procedure is

adopted here and some necessary modifications are also done for inclined pile under tensile load.

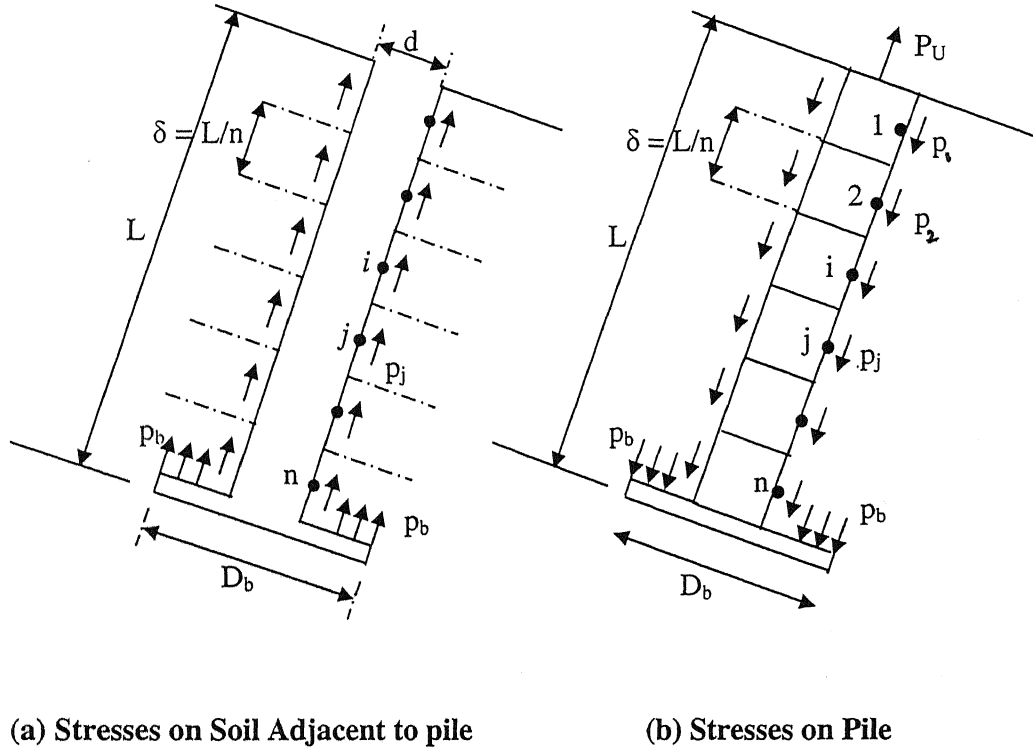


Fig. 3.2 Discretization Details

The complete discretization details are shown in Fig. 3.2. The pile is divided into n elements of equal length. The nodes of the elements are considered at their centers. Each element is acted upon by uniform shear stress p_j . The base is assumed to be rigid and is subjected to uniform vertical stress p_b .

The stresses acting on the pile is shown in Fig. 3.2 (a). Fig. 3.2(b) shows the stresses acting on the soil adjacent to the pile. Due to the application of the uplift load P there are $n + 1$ stresses (including p_b) and the same number of displacements. The problem is to

evaluate these two quantities. These are evaluated by equating the pile displacement at all the nodes to the corresponding displacement in the soil. The following steps are adopted to evaluate these.

1. First find the soil displacements in terms of stresses at all the nodes. In evaluating the soil displacements, the unknown force on each element was resolved into vertical and horizontal components, and vertical and horizontal displacements due to each of these components were calculated using Mindlin's equation. These displacements were then combined to give the axial displacement.
2. Find the pile displacements in terms of stresses at all the nodes.
3. In order to satisfy the compatibility condition between soil and pile displacements, equate the soil and pile displacement at all $n+1$ nodes. This gives rise to $n+1$ equation with $n+1$ unknown stresses.
4. Solve these equations for the unknown stress, p_i .
5. Substitute back either in soil displacement equations or on pile displacement equations to evaluate $n+1$ unknown displacement.
6. The load distribution along the pile shaft is then evaluated from these known stresses.

The above steps are explained in details in the following sections. Analysis for local yield of soil is further taken care of by modifying the above approach incorporating the suggestions made by Poulos and Davis (1968). The same will be discussed as and when required to take into account the pile-soil slip.

3.4.1 Soil Displacement Equation:

Considering a typical element i in Fig. 3.2, the displacement of the soil adjacent to the pile at i resulting from the stress p_j on an element j can be expressed as

$${}_s\rho_{ij} = -\frac{d}{E_s} I_{ij} p_j \quad \dots\dots\dots (3.1)$$

Where I_{ij} = displacement factor for element i due to stress at j

As a result of all elements and base the soil displacement at i is

$${}_s\rho_i = \frac{d}{E_s} \sum_{j=1}^n (I_{ij} p_j) + \left(\frac{D_b}{E_s} I_{ib} \right) p_b \quad \dots\dots\dots (3.2)$$

Where I_{ib} = displacement factor for element i due to stress at base

Similarly, the displacement of the soil directly beneath the pile base may be expressed as

$${}_s\rho_b = \frac{d}{E_s} \sum_{j=1}^n (I_{bj} p_j) + \left(\frac{D_b}{E_s} I_{bb} \right) p_b \quad \dots\dots\dots (3.3)$$

Therefore, the displacement of all elements of the pile may be conveniently written in matrix form as

$$\{ {}_s\rho \} = -\frac{d}{E_s} [I_s] \{ p \} \quad \dots\dots\dots (3.4)$$

In which, $\{ {}_s\rho \}$ = soil-displacement vector

$\{ p \}$ = pile stress vector

$$[I_s] = \begin{bmatrix} I_{11} & I_{12} & \cdot & \cdot & \cdot & I_{1n} & I_{1b} \\ I_{21} & I_{22} & \cdot & \cdot & \cdot & I_{2n} & I_{2b} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ I_{n1} & I_{n2} & \cdot & \cdot & \cdot & I_{nn} & I_{nb} \\ I_{b1} & I_{b2} & \cdot & \cdot & \cdot & I_{bn} & I_{bb} \end{bmatrix} \quad \dots\dots\dots (3.4a)$$

$[I_s]$ is the $(n+1)$ by $(n+1)$ matrix of soil displacement influence factors, all elements of this matrix being obtained by double integration of the Mindlin's Equation as described in Appendix.

The unknown force on each element is resolved into vertical and horizontal component, and vertical and horizontal displacements caused by each of these components are calculated by Mindlin's equation for the displacement caused by a point load within a semi-infinite mass. These displacements are then combined to give the axial displacement.

3.4.2 Pile Displacement Equation:

For Inextensible Pile:

The displacement $\{p_p\}$, of pile assumed rigid can be written as

$$\{p_p\} = -[I] \cdot \lambda \dots\dots\dots (3.5)$$

Where, $[I]$ is a unit vector, and, λ the axial displacement of pile.

Equilibrium of forces requires

$$P = \frac{\pi d L}{n} \sum_{j=1}^n p_j + \frac{\pi}{4} (D_b^2 - d^2) p_b \dots\dots\dots (3.6)$$

The compatibility of displacements is satisfied by equating $\{p_p\}$ and $\{s_p\}$ i.e. Equation (3.4) and (3.5), from which we can write,

$$\{p\} = -\frac{E_s}{d} [L_s]^{-1} \{1\} \cdot \lambda \dots\dots\dots (3.7)$$

Substituting Eq. (3.7) into Eq. (3.6), the pile displacement λ , is obtained as

$$\lambda = \frac{-P}{E_s d} I_t \dots\dots\dots (3.8)$$

where I_t is the influence factor due to tensile load.

3.5 Modification to Basic Analysis for Pile-Soil Slip

The analyses described above require that no slip occurs at the pile soil interface. It is assumed that the soil is an ideal elastic material capable of resisting any shear stresses which may be developed between the pile and the soil. However, real soil has finite shear strength and the interface between the pile and the soil has finite shear strength. When the shear stress reaches the adhesive strength slip occur between the pile and the soil. Thus the solutions presented are valid only while conditions in the soil remain elastic and the shear stresses along the pile are less than the adhesive strength.

By using the method similar to that described by D'Applonia and Romualdi (1963), Poulos and Davis (1968) and Mattes and Poulos (1969), the elastic analysis can be modified to take account of possible slip, provided that the following assumptions are made:

1. Local yield or slip occurs at the pile-soil interface when the average shear stress on any pile element, calculating from the elastic analysis, reaches the limiting value τ_a .
2. Although compatibility of pile and soil displacements at a yielded element is no longer possible, displacements anywhere in the soil caused by the limiting stress τ_a are still given by elastic theory.

3. Failure of the tip or the base of the pile occurs when the base pressure reaches the ultimate bearing capacity of the base, the displacements of the soil elsewhere resulting from this pressure still being given by elastic theory.

However, for tensile loading as the concerned pile is being lifted base pressure can take place and in the analysis it is not taken into account.

Here the only case considered is that of an inclined anchor pile in a dry sand medium. For an inextensible pile composed of $n + 1$ elements (including the base) elastic conditions prevail until the load carrying capacity of the most heavily loaded element of the pile is reached. If slip occurs along the pile shaft it will be when

$$p_{\max} = K\sigma_v' \tan \delta \dots\dots\dots (3.9)$$

Where p_{\max} is the maximum shear stress on the pile shaft, K is the lateral earth pressure coefficient, σ_v' is the average overburden pressure over the embedded length of the pile and δ is the angle of friction between the pile and the soil.

Broms (1966) recommended the values of K shown in the Table 3.1 for piles driven into sand.

Table 3.1 Values of K

Pile Material	Values of K	
	Loose Sand	Dense Sand
Steel	0.5	1.0
Concrete	1.0	2.0
Timber	1.5	4.0

If failure of the anchor occurs the stress on the base is

$$P_{bmax} = N_q \sigma_v' \dots\dots\dots (3.10)$$

Where N_q is the bearing capacity factor and according to Berezantzev et al (1961) is a function of angle of shearing resistance ϕ and L/d ratio of the pile.

If the load applied to the pile increases beyond the value required to cause the first slip or yield, the element which has failed can take no additional load and the increase in load is thus redistributed between the remaining elastic elements until the load carrying capacity of the next most heavily loaded element is reached.

This stage may be analysed by calculating from the original elastic soil displacement matrix (Equation 3.4a) the displacements at the centers of the remaining elements due to the unknown stresses on the elastic elements and the known ultimate stress on the element which has slipped. A set of n equations is thus obtained from previous $n + 1$ equations and these n equations are solved to give the stress distribution on the pile and the displacement of the pile top until slip of the next element occurs.

This procedure is repeated until all the elements in the pile including the anchor base have failed. At each of the load increments, the corresponding displacements of the top of the pile are determined and thus the load-displacement relationship up to failure is obtained.

CHAPTER 4

EXPERIMENTAL SET-UP AND TEST PROCEDURE

4.1 Experimental Set-up

The schematic diagram of test setup, loading arrangement and details of the model inclined anchor pile are shown in Fig 4.1.

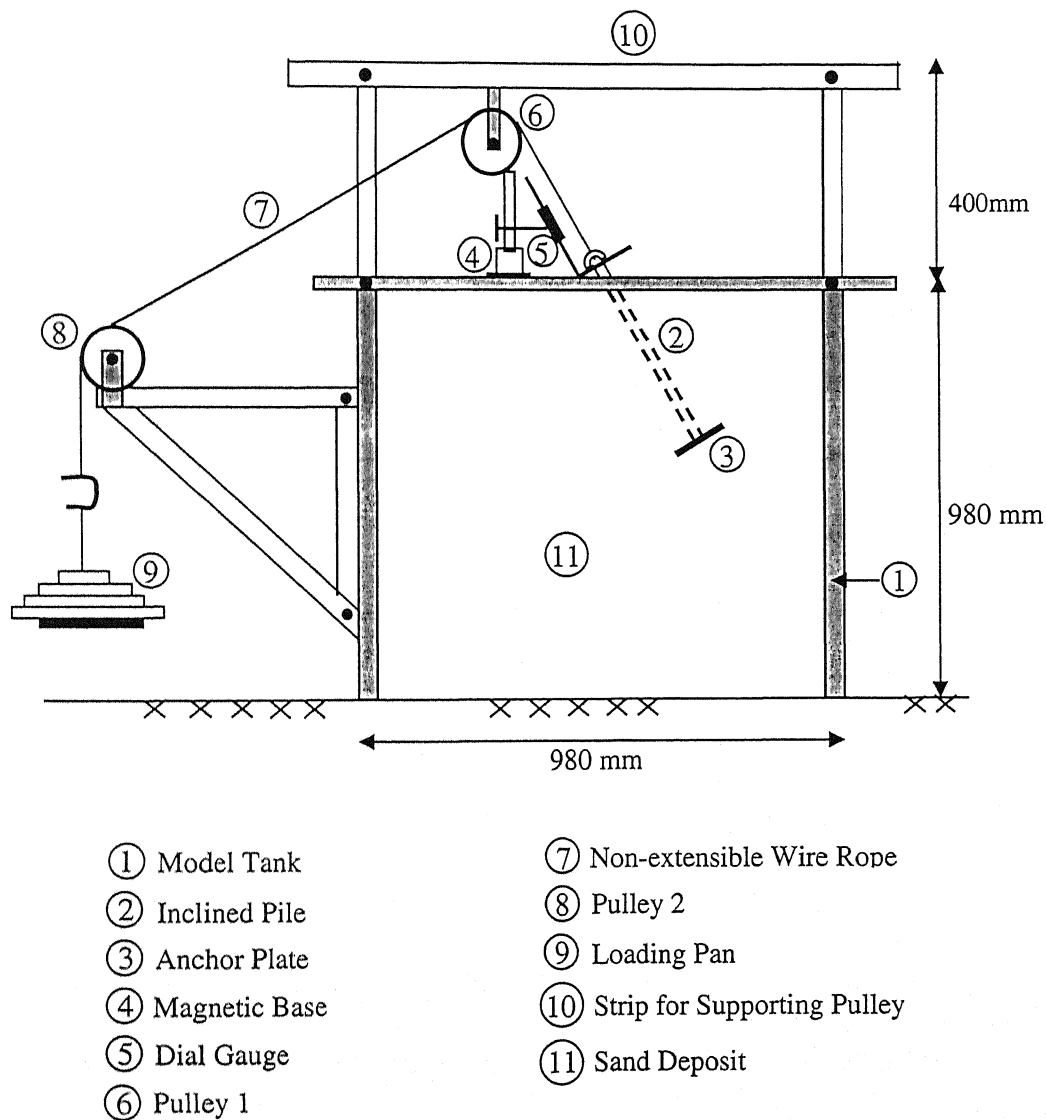


Fig 4.1 Details of Experimental Setup

4.1.1 Model Tank

A model steel tank of dimension 980mm X 980mm X 970mm is used in the present test program to carry out the laboratory test. The pile is placed into the tank by providing a minimum lateral clearance of $10d$ to satisfy the minimum requirements to avoid the effect of container wall on the test results.

4.2.2 Model Anchor Piles

In the present investigation, model piles are made up of mild steel rods of square cross section of size 20mm X 20 mm. Piles are tested at two different embedment depths of 40 cm and 60 cm such that the length (L) to diameter (d) ratio (L/d) becomes 20 and 30 respectively. Anchor at the bottom of the pile shaft is provided by means of square plate of 10 mm thickness. Two different plate sizes of 40 mm and 60 mm are used so that the diameter of the base (D_b) to the diameter of pile shaft (d) ratio, i.e. D_b/d ratios obtained are 2 and 3. The plate is fastened to the shaft bottom by means of thread and bolt arrangement. Soil-pile interface friction angle δ is measured by shear box test for MS pile surface and sand of desired density. Soil-pile interface angle is observed to be 28° for dense sand. Original pile surface is used without any distortion, and piles can be said to be smooth.

4.2.3 Pulleys

Pulleys of cast iron of capacity not less than 200 kg are used to apply uplift load with the help of flexible wire rope of 2 mm diameter. Double pulley arrangement is prepared to apply the uplift loads. Wire rope is connected to the center of area of pile cap through U hook, to eliminate the eccentricity in application of load as far as practicable. Pulleys used in the present experiments are friction-less pulleys for all

practical purposes, and hence the entire load applying at the loading pan can be assumed to be transferred to the pile and finally to the sand medium.

4.2.4 Dial Gauges

Two dial gauges of 0.01 mm sensitivity and 50 mm capacity in conjunction with magnetic base plate are used to record the displacement of the pile under the pullout loads. The dial gauges are placed over the aluminium strips attached to the pile cap. It is ensured that the both the dial gauges are equidistant from the center of the pile cap.

4.2.5 Hoppers

Hoppers made up from Aluminium sheets with slots opening of 4 mm size. They are utilized to pour sand by rainfall technique from a certain height to achieve the required density. A thread is connected to the bottom of the hoppers to maintain the constant height of fall of sand.

4.2.6 Penetrometer

Penetrometer is used to check the accuracy and the uniformity of the density of the deposited sand during the experiment. Dynamic Penetrometer consists of a circular rod of 9.5 mm diameter and a drop weight of 2.85 kg with height of fall of 30 cm. Penetration per blow is recorded for each experiment. Dynamic penetration tests are carried out after the sand filling and before the commencement of the experiment, to comment on consistency of the density of deposition and its uniformity. Dynamic penetration tests are also performed after the completion of the experiments near the corner of the tank, to evaluate the effects of pile loading on density of sand deposition. It is found that the penetration values before the experiment and after

experiment are almost same. This implies that the pile loading has negligible effect on sand density near the tank corners and tank dimension are quite large as compared to the zone of influence around the pile, and hence the pile behaviour is practically unaffected by any interference due to tank boundaries. On an average two to three numbers of blows are recorded for penetration of 30 cm and six to eight blows are required for penetration of 60 cm.

4.2.7 Test Medium

Dry Ennore sand has been used as foundation medium in the tank to perform the test. Sand is poured in the test tank to a dense state by hoppers using rainfall technique. In addition to the main testing program, routine tests like direct shear test; compaction test, sieve size analysis etc are performed to determine the properties of the sand used in the experiments

4.2 Testing Procedure

All the axial uplift tests for the inclined anchor piles are conducted in the steel tank (shown in Fig 4.1) filled with dry Ennore sand as foundation medium. Pouring of sand has been done by means of standard rainfall technique using the slot hoppers maintaining a constant height of sand fall of 40 cm to obtain uniform and constant density for all the fillings. The piles are kept in positions at the desired inclination (15^0 , 30^0 and 45^0) by means of a longitudinal split pile cap. The pile cap with pile at fixed inclination is supported on flat surface running across the width of the tank. Sand pouring has been continued till at least 1 cm of pile is exposed for insertion of a ball and socket arrangement with wire rope at the top of the pile.

Axial pulling loads are applied on the pile top at desired angle by a wire, one end of which is attached to the pile top through the ball and socket arrangement and the other to a loading pan which accepted dead weights. The wire is taken first over an inverted adjustable pulley with frictionless ball bearing and then over the second fixed pulley. Magnetic base dial gauges having sensitivity 0.01 mm are used to measure the axial displacement of the pile. Two dial gauges making an angle same as the pile inclination are placed on the pile cap by using aluminium strips at 180^0 apart and equidistant from the load axis.

Tests are conducted with piles of two different embedment lengths of 40 cm and 60 cm (respective L/d ratios are 20 and 30) and for each type of piles, two different anchor plates of size 4cm x 4cm and 6cm x 6cm are used. Each set of anchor piles are placed at three different inclinations (15^0 , 30^0 and 45^0) and vertically also.

Axial uplift loads in increment are applied on piles up to failure and corresponding axial movements of the top of the piles are measured. The uplift capacity of the pile is taken as the load at which the pile is pulled out.

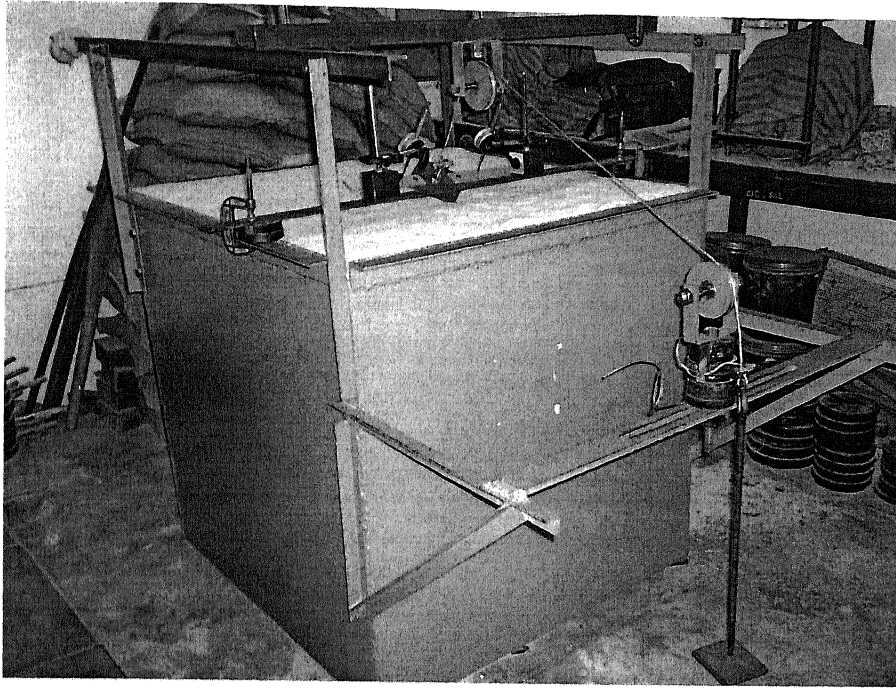


Plate 4.1 Test Set-Up for Inclined Anchor Pile

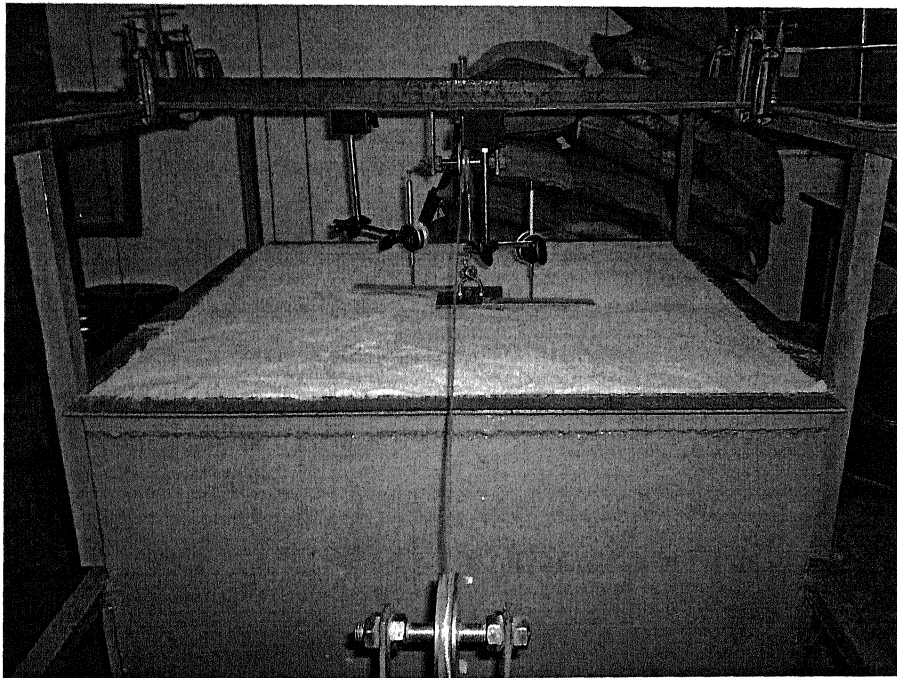


Plate 4.2 Test Set-Up for Vertical Anchor Pile

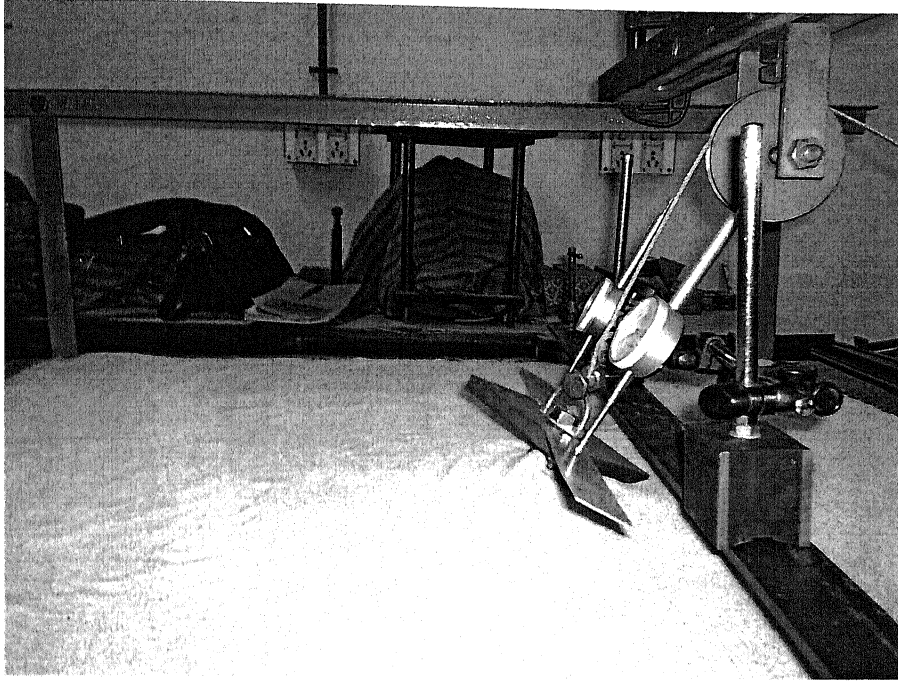


Plate 4.3 Dial Gauge Arrangement for Inclined Piles

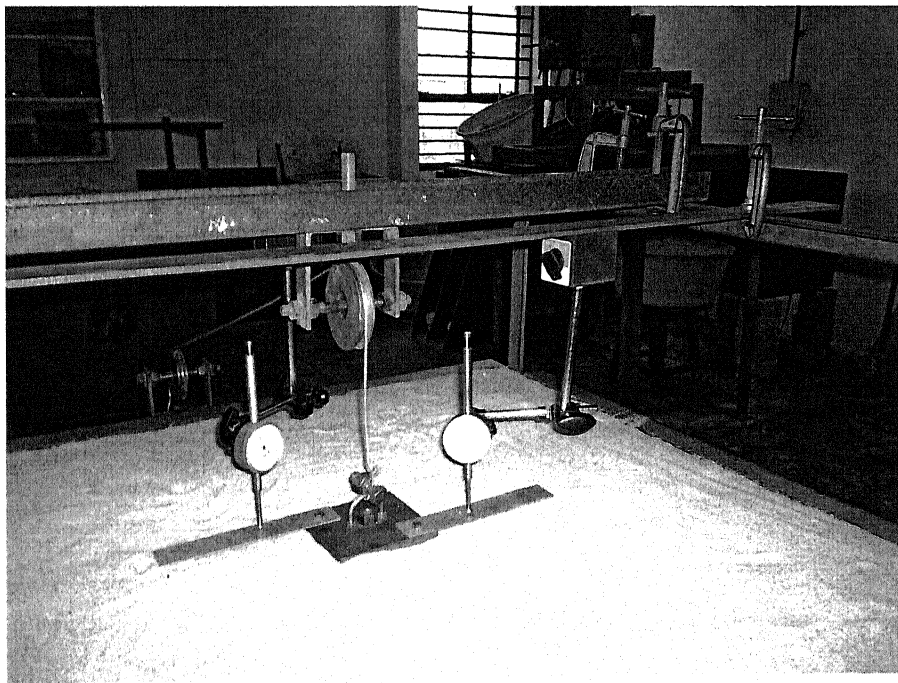


Plate 4.4 Dial Gauge arrangement for vertical piles

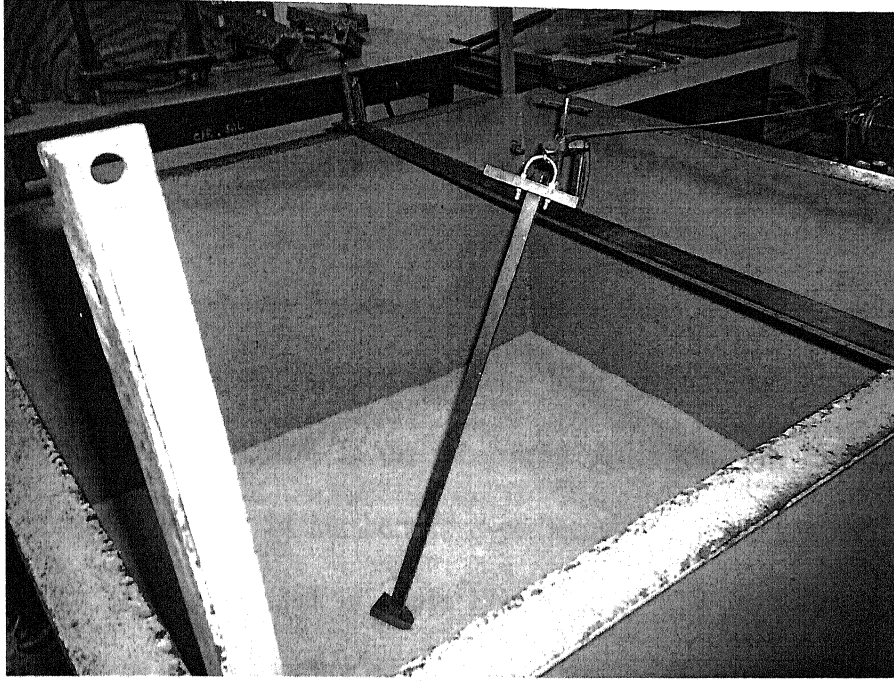


Plate 4.5 Fixing of Anchor Pile at a Particular Inclination

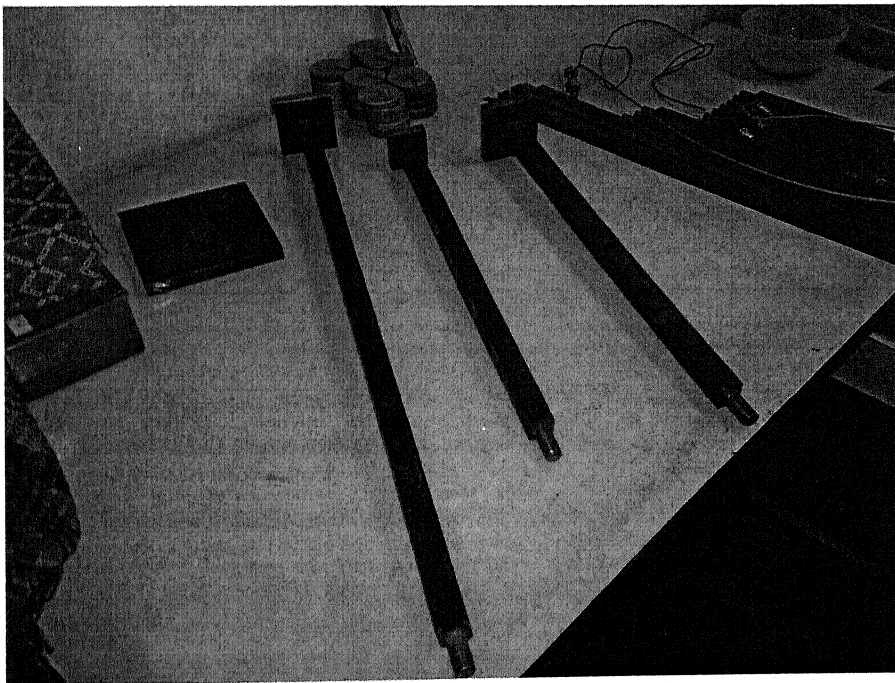


Plate 4.6 Model Piles with Anchors

CHAPTER 5

RESULTS, COMPARISONS AND DISCUSSIONS

5.1 General

The results obtained from the proposed method of analysis by using the elastic approach for a single inclined anchor pile under axial uplift force in a dry sand medium are presented in details in the subsequent paragraphs. An experimental program consisting of total 24 numbers of experiments was performed on model inclined anchor piles embedded in dense Ennore sand. The load displacement characteristics of the inclined anchor piles obtained from the experimental results are also presented in this chapter. To check the correctness of the predicted values by the present analytical method comparisons of results are also provided. To validate the present analytical solutions, comparisons presented hereby are classified in two categories:

- a) Comparison of present analytical study with the data obtained from the past investigations; and
- b) Comparisons of present experimental results with the present analytical solutions.

The results are presented in three categories.

1. First, a series of dimensionless parametric solutions is presented for the stress and load distribution in a pile, and for the settlement of a single pile in a homogeneous soil mass using the proposed elastic analysis. The proposed method is validated by comparing the solutions with the available data from past investigations.

2. Modifying the proposed method by considering the pile soil slip along the pile, load displacement behaviour for inclined anchor pile under uplift load is presented and also compared with the present experimental data.
3. Parametric study has been carried out by the proposed method and comparative study of ultimate uplift capacity of inclined anchor piles for different L/d ratios; different inclinations of pile and with different diameters of anchors are presented.

5.2 Analytical Solutions for Stress and Load Distribution

The solutions provided in the subsequent paragraphs are for purely elastic behaviour of an inclined anchor pile in homogeneous soil mass having constant Young's Modulus and Poisson's Ratio. In obtaining the solutions the pile has been divided into 10 elements. Investigation into the sensitivity of the solutions on the number of elements used in the analysis have shown that the use of 10 elements to divide the pile shafts leads to answers of acceptable accuracy unless the pile is too long ($L/d > 50$), in which case 20 elements may be desirable. In Fig 5.1, for a 30° inclined pile ($L/d=20$) with anchor base ($D_b/d=2$), the solutions obtained from Equation 3.7 to get the stress distribution along the pile surface for 10 and 20 elements are shown. It is observed that using large number of elements higher shear stresses being obtained near the base, but the variation is not more than 10%. The curves become smoother only by using more elements. It was also found that the displacements obtained by the use of 10 elements differed by less than 5% from those obtained using 20 elements. As the emphasis lay with the displacements rather than with the details of the shear stress distribution, the above sacrifice in accuracy in using 10 elements is considered justified in view of the saving in time.

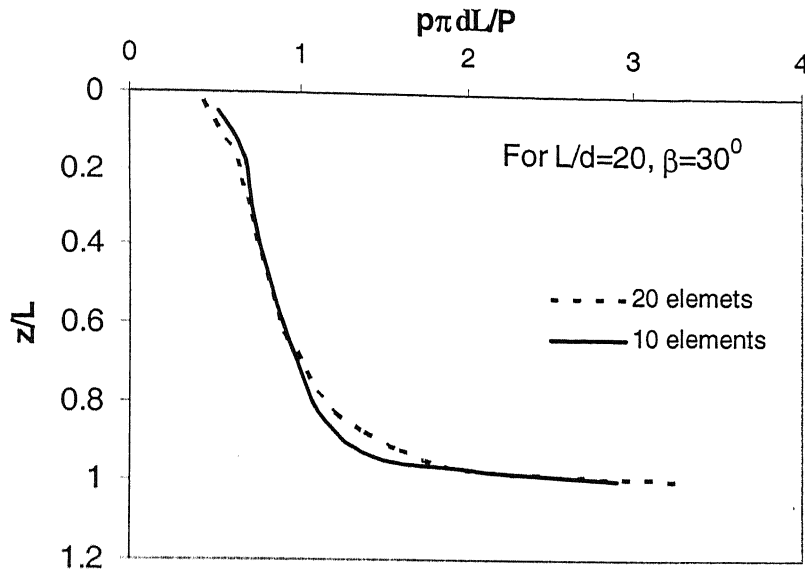


Fig 5.1 Distribution of Side Shear with Pile Proportion Using 10 and 20 Elements

5.2.1 Load Transfer along Pile

Equation 3.7 has been solved to obtain the distribution of side shear along the pile in a semi-infinite soil mass. Typical distributions of shear stress on the pile shaft are shown in Fig 5.2 for the case when Poisson's Ratio (ν_s) is 0.2 and pile inclination is 30° with the vertical. Anchor base diameter is considered 2 times the shaft diameter. The general change in the shear stress distribution with increasing L/d ratio can be readily observed from Fig. 5.2. For shorter piles ($L/d = 5$) there is a concentration of shear stress near the ends of the pile and a minimum value near the center. For slender piles, the shear stress increases gradually from a minimum near the top of the pile to a maximum near the base of the pile. Near the tip ($z/L = 1$), the values of shear stresses are very high being more than 4-5 times the average value. With increasing L/d ratio, the shear stresses tend to become uniform over a large part of the pile.

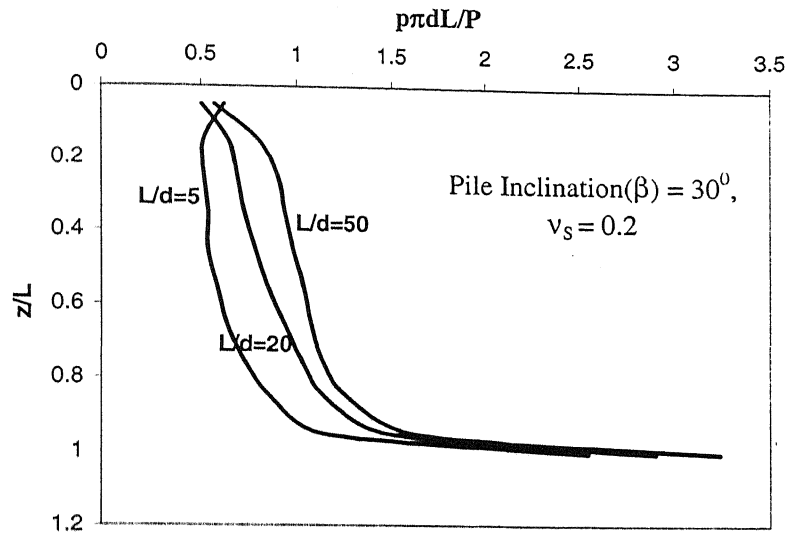


Fig 5.2 Variation of Distribution of Side Shear with Pile Proportion

Using the same computer program, the distribution of side shear for a vertical pile without anchor has been obtained and to validate the program the same is compared with the shear stresses obtained by Madhav and Poorooshab (1988) in Fig 5.3. They had analysed the shear stresses along the vertical pile shaft under tensile load for $\nu_s = 0.5$. The comparison is done for pile having $L/d = 25$. From the Fig 5.3, it has been observed that the shear stress distribution along the pile shaft obtained by the present study is almost identical to that obtained by Madhav and Poorooshab (1988).

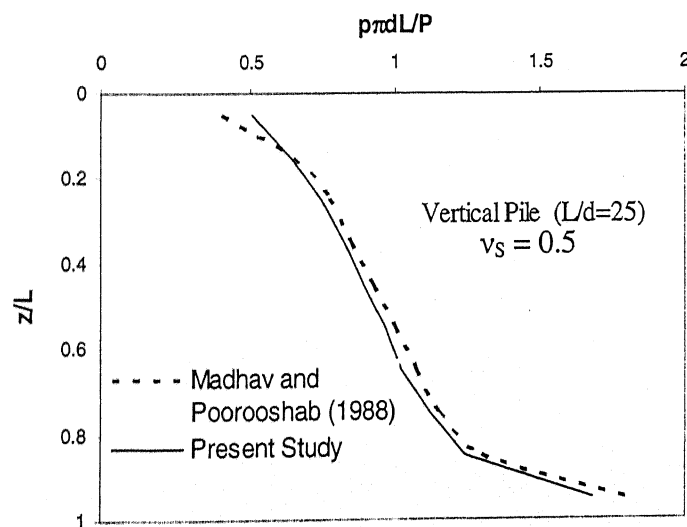


Fig 5.3 Comparison of Side Shear Stress with Past Investigations

The distribution of side shear along the pile shaft for different pile inclination for a particular L/d ratio ($=20$) and D_b/d ratio ($=2$) is shown in Fig 5.4. As the pile inclination increases the stresses generated along the pile also increases.

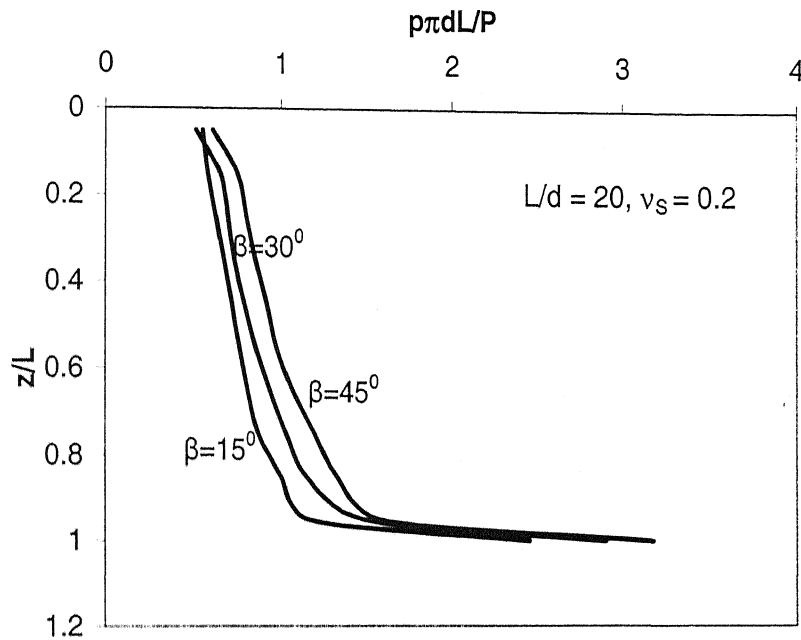


Fig 5.4 Distribution of Side Shear for Different Pile Inclination

The proportion of the total load carried by the enlarged base or the anchor is plotted against the L/d ratio in Fig 5.5.

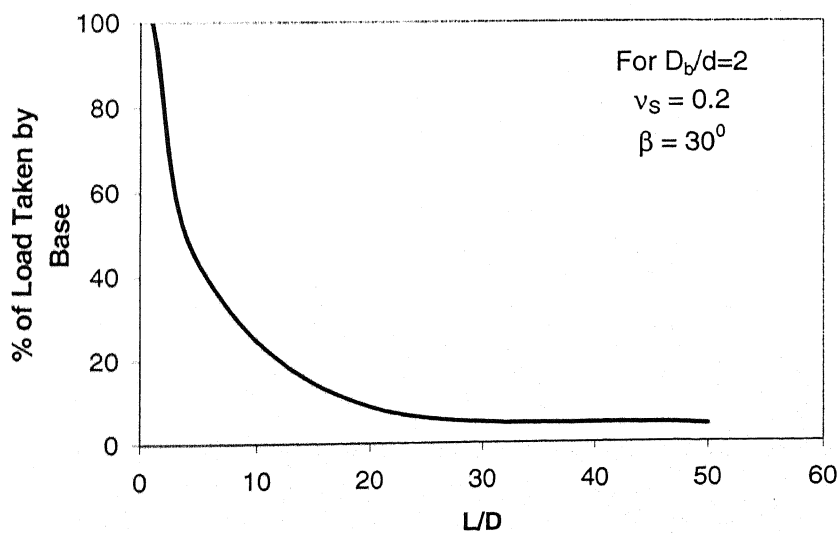


Fig 5.5 Variation of Load Taken by Base with L/d Ratio

It can be seen from Fig 5.5 that as L/d ratio increases there is a sharp decrease in the percentage load taken by the base and the decrease occurs with a decreasing rate with L/d ratio, till it becomes negligible beyond which there is no further reduction in its' value.

In Fig 5.6, the variation of load taken by the base with the ratio D_b/d is shown for an inclined pile in a semi-infinite mass. As the diameter of the pile base increases the load taken by the base increases as expected. For smaller values of L/d , the more rapid is the rate of increase of base load with increasing base diameter. With the values L/d smaller than about 5 it is apparent that even a relatively small enlargement of the base results in the pile becoming effectively a spread footing with almost all the load taken by the base. For slender pile enlargement of the base has little influence on the small amount of load transferred to the base.

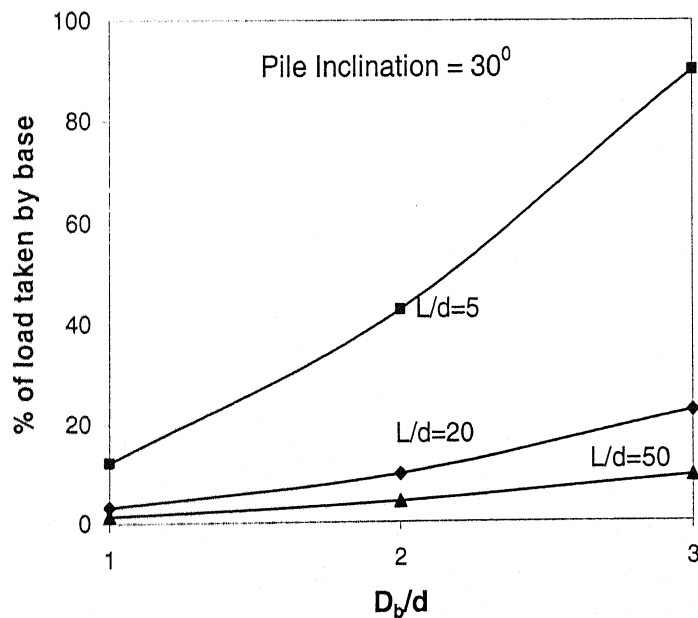


Fig 5.6 Effects of Enlarged Base or Anchor on Base Load

5.2.2 Pile Displacement Factor

From Equation 3.8 the pile displacement caused by the pullout force can be conveniently expressed as dimensionless influence factors I_t . The original pile displacement, λ can be found out from the following expression:

$$\lambda = \frac{P}{E_s d} I_t \quad \dots\dots\dots (5.1)$$

Where P is the applied uplift load, d is the pile diameter and E_s is the elastic modulus of the soil mass. The values of the influence factors are plotted against a wide range of L/d ratios for two values of ν_s , 0.2 and 0.5 in Fig 5.7.

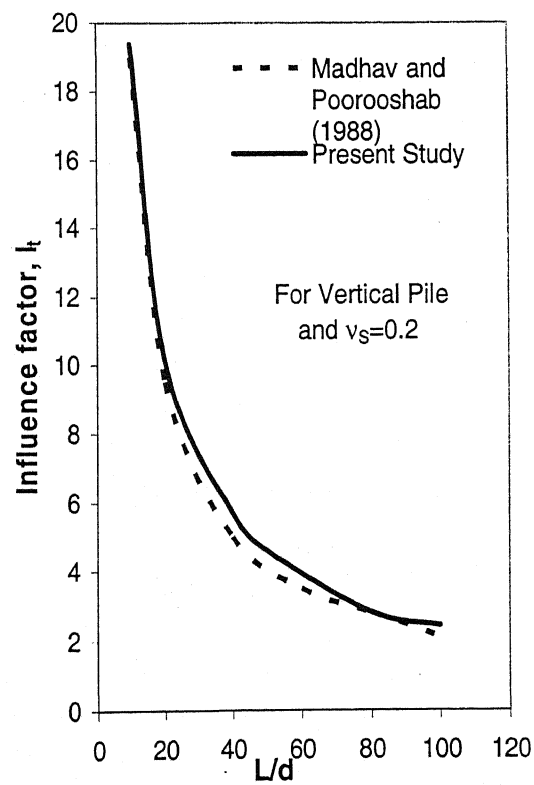
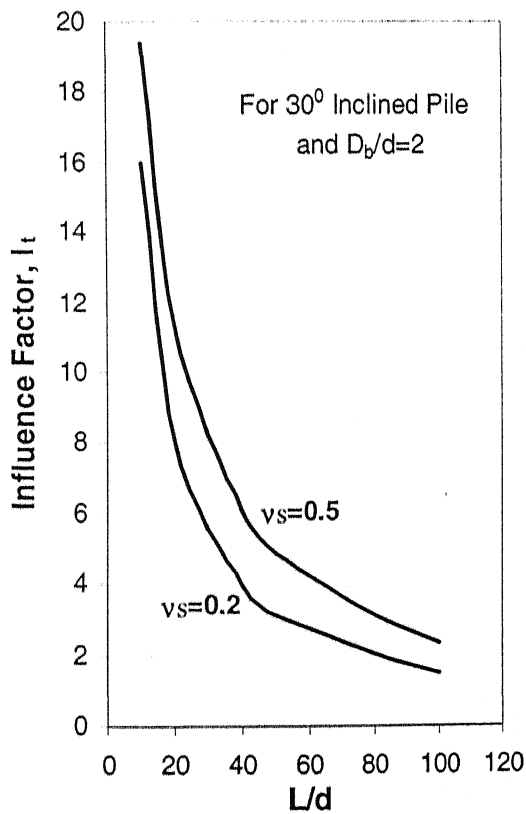


Fig 5.7 Displacement Influence Factor

Fig. 5.8 Comparison with Past Results

From Fig 5.7, it is observed that the value of displacement influence factor, I_t , decreases with the increasing L/d ratios of the pile, as anticipated. Longer the pile, the smaller will be the displacement. Using the same analysis, the displacement influence factor for a vertical pile without anchor is plotted in Fig 5.8 and compared the same with the values of displacement influence factors as obtained by Madhav and Poorooshab (1988).

5.3 Experimental Results and Comparisons of Measured Values with Computed Values from Analytical Solutions

An experimental program consisting of total 24 numbers of experiments was performed on model inclined anchor piles embedded in dense Ennore sand in the model tank to validate the present analytical results. Two different types of L/d ratios (20 and 30) of the piles are considered. For each L/d ratio, three different D_b/d ratios are taken, one with D_b/d ratio equal to 1, i.e. conventional pile without base enlargement, another with D_b/d ratio equal to 2 and other with D_b/d ratio equal to 3. Each set of anchor piles have been tested with 3 different inclinations with the vertical 15° , 30° and 45° and vertically also.

In addition to the main testing schedule, some routine tests like shear box test, tri-axial test, sieve analysis and compaction test are also carried out in order to determine the basic characteristic of the sand medium and pile material. The Ennore sand used as test medium in the whole experimental program has the following properties:

- $D_{60} = 0.7$ mm, and $D_{10} = 0.62$ mm
- Uniformity coefficient, $C_u = 1.1$ and Coefficient of curvature, $C_c = 1.0$; the soil is uniformly graded (poorly graded) sand (SP).

- Specific Gravity = 2.67
- Maximum and minimum void ratio, $e_{\max} = 0.896$ and $e_{\min} = 0.577$
- Maximum and minimum dry density, $\gamma_{\max} = 1.663 \text{ gm/cc}$ and $\gamma_{\min} = 1.414 \text{ gm/cc}$
- The placement density and void ratio during the test, $\gamma = 1.6 \text{ gm/cc}$ and $e = 0.65$
- Relative density during test, $D_R = 77.6 \%$
- Corresponding angle of shearing resistance, $\phi = 39^\circ$
- Soil pile interface friction angle, $\delta = 28^\circ$.
- Modulus of elasticity of sand = 400 Kg/cm^2
- Modulus of elasticity of pile material = $2 \times 10^6 \text{ Kg/cm}^2$

The findings of the experimental program are described in the following sections

5.3.1 Load Displacement Characteristics from Experimental Results

Uplift loads are applied axially on the inclined anchor pile in increments and for each applied load axial displacement of the pile are measured by the two dial gauges. Loads are applied till failure; the uplift capacity of the pile is taken as the load at which the pile is pulled out. Axial uplift load versus axial movement of inclined piles have been presented through Fig 5.9 to Fig 5.14.

The general trend of all load displacement curve is more or less same. At the initial stage of the loading the load displacement behaviour is almost linear, which gradually deviates to non-linear response with increasing of load.

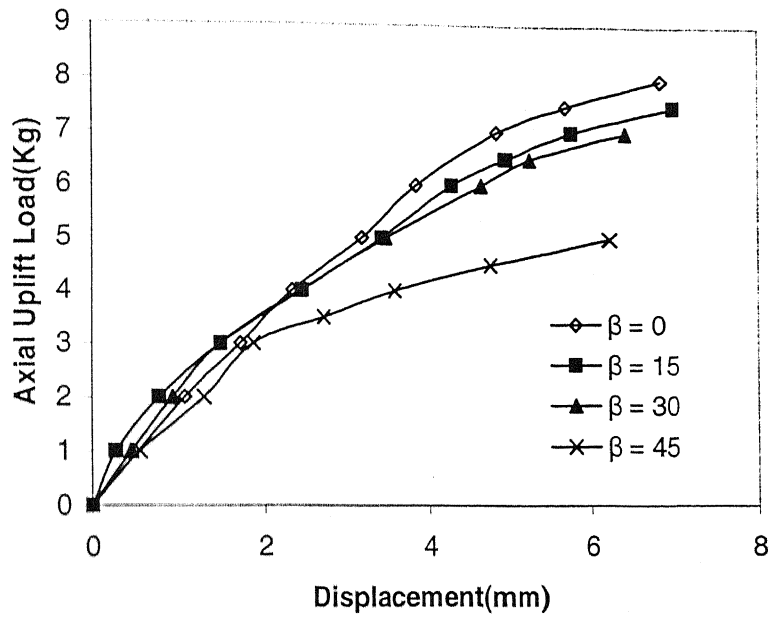


Fig 5.9 Uplift Load Vs Axial Movement of Pile
($L/d = 20$, $D_b/d = 1$)

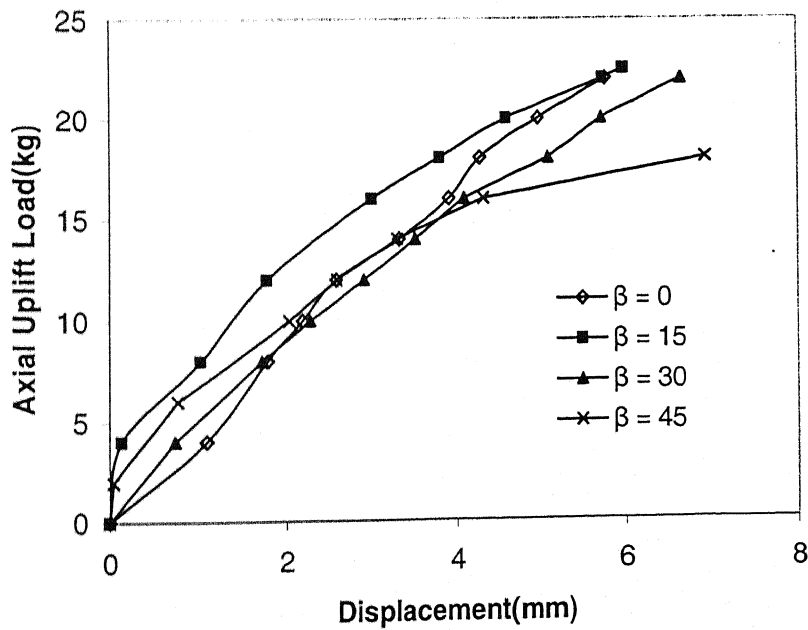


Fig 5.10 Uplift Load Vs Axial Movement of Pile
($L/d = 20$, $D_b/d = 2$)

दुखोत्तम काशीनाथ केलकर पुस्तकालय
भारतीय प्रौद्योगिकी संस्थान कानपुर
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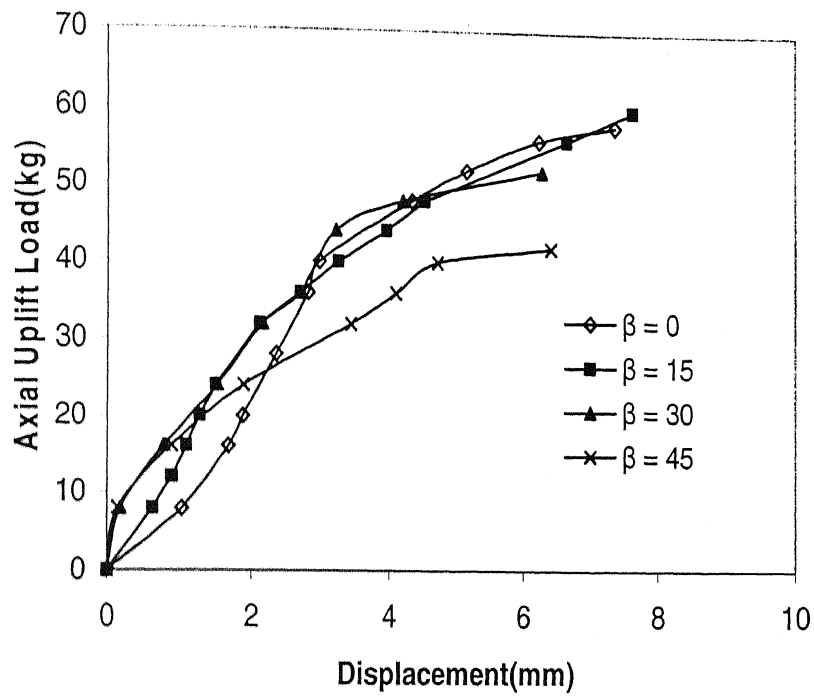


Fig 5.11 Uplift Load Vs Axial Movement of Pile
($L/d = 20$, $D_b/d = 3$)

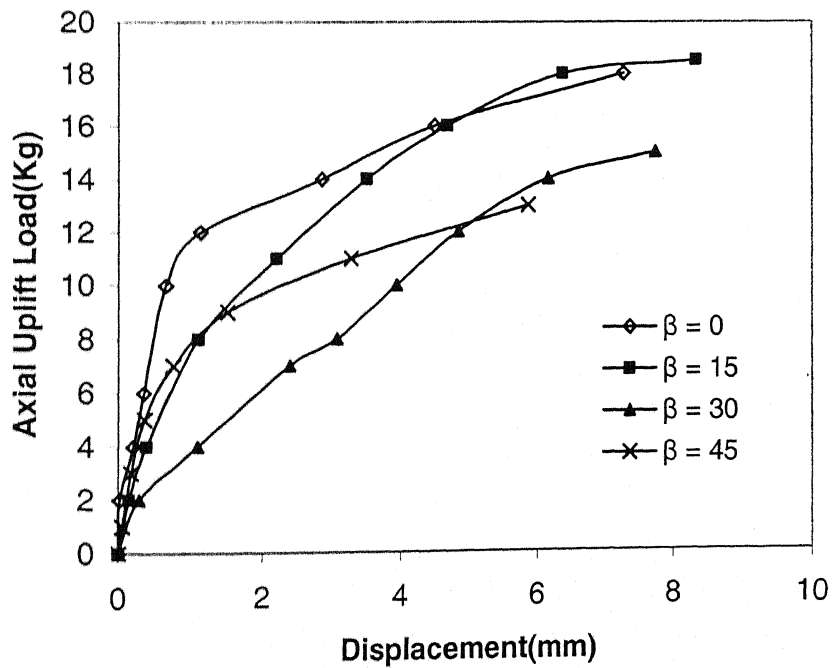


Fig 5.12 Uplift Load Vs Axial Movement of Pile
($L/d = 30$, $D_b/d = 1$)

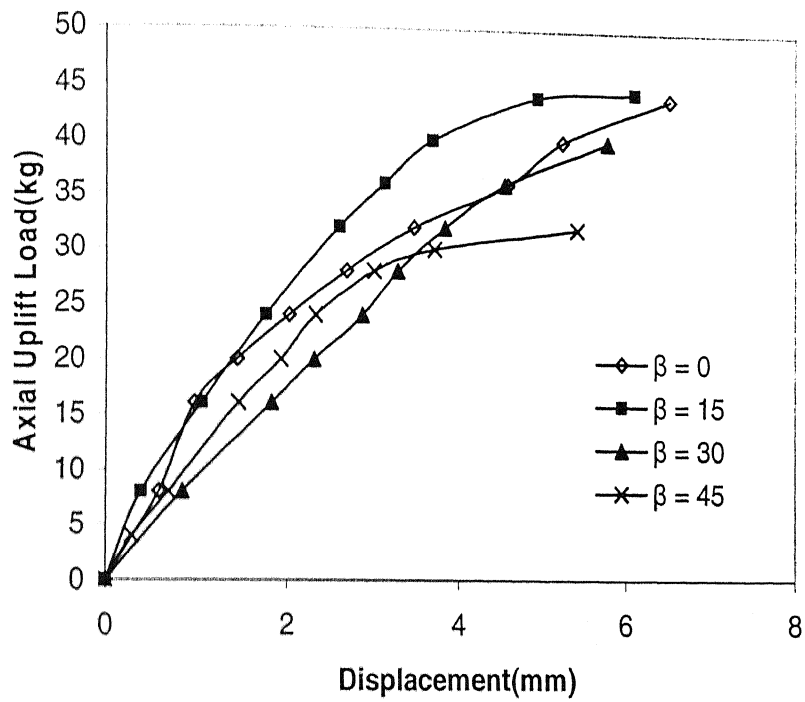


Fig 5.13 Uplift Load Vs Axial Movement of Pile
($L/d = 30$, $D_b/d = 2$)

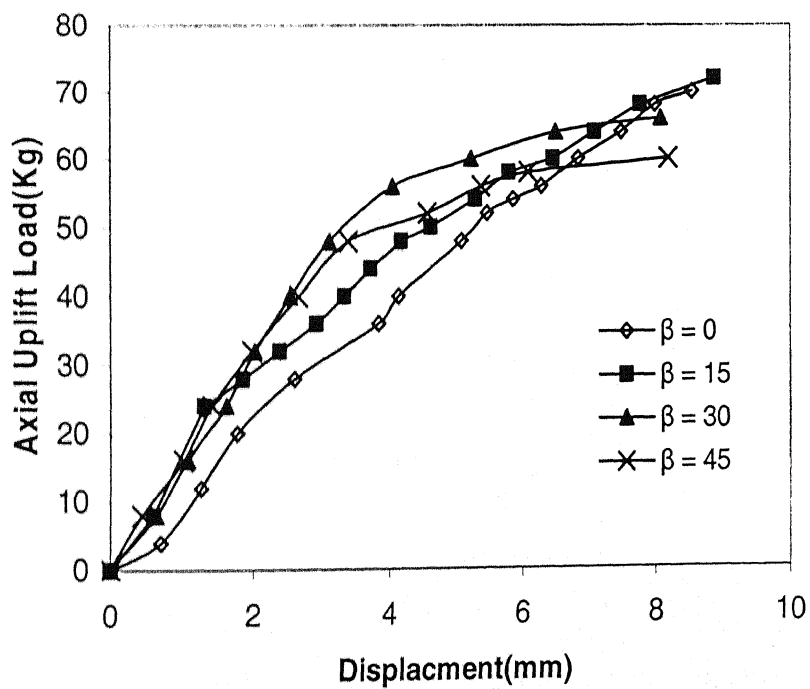


Fig 5.14 Uplift Load Vs Axial Movement of Pile
($L/d = 30$, $D_b/d = 3$)

5.3.2 Comparison of Experimental Load Displacement Curves with Analytical Solutions

To validate the present analytical study on the uplift capacity of the inclined anchor piles based on the elastic approach, the analytical results are compared with the experimentally observed results. The basic elastic analysis is modified taking into account the slip along the pile soil interface as described in section 3.5. The axial movements of the pile due to the axial uplift load are determined for each increment of load until the stresses of each element and the base reach the limiting values. The load displacement curves are then plotted.

The load displacement curves plotted by analytical solutions are compared here with some of the experimental results in the following figures.

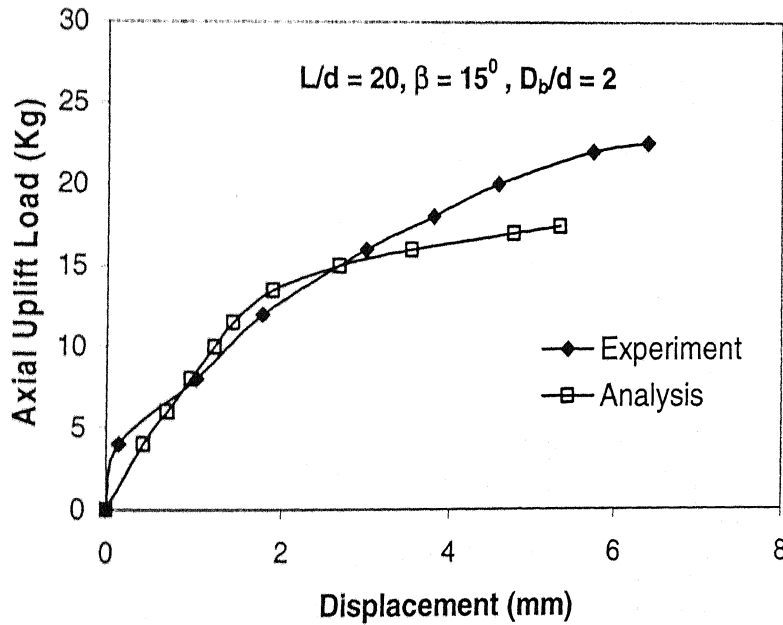


Fig 5.15 Comparison of Load Displacement Curve
($L/d = 20$, $\beta = 15^\circ$, $D_b/d = 2$)

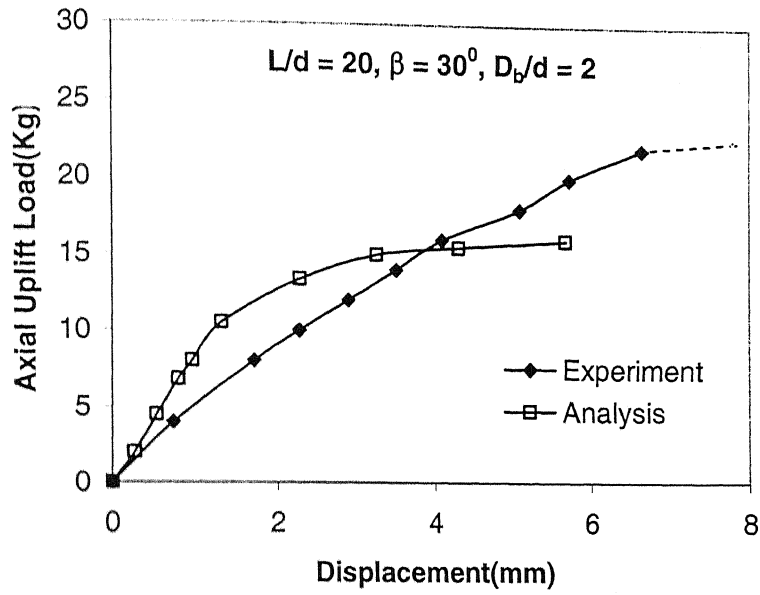


Fig 5.16 Comparison of Load Displacement Curve
($L/d = 20$, $\beta = 30^\circ$, $D_b/d = 2$)

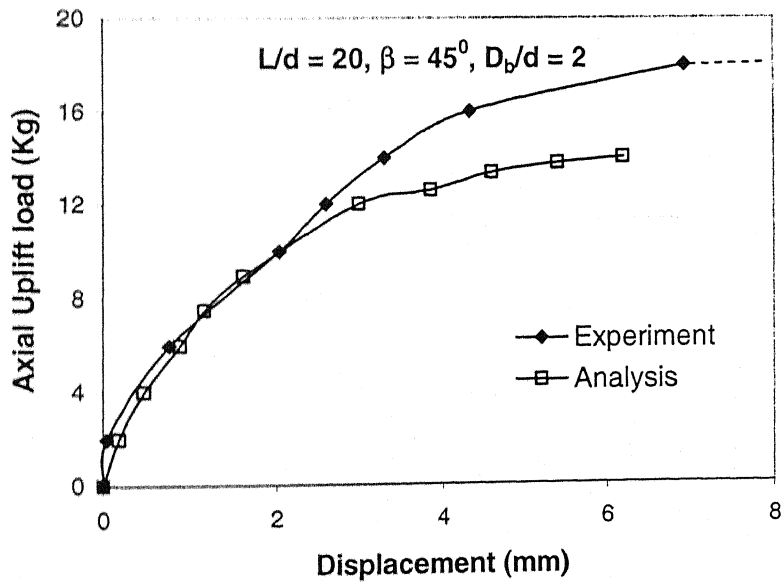


Fig 5.17 Comparison of Load Displacement Curve
($L/d = 20$, $\beta = 45^\circ$, $D_b/d = 2$)

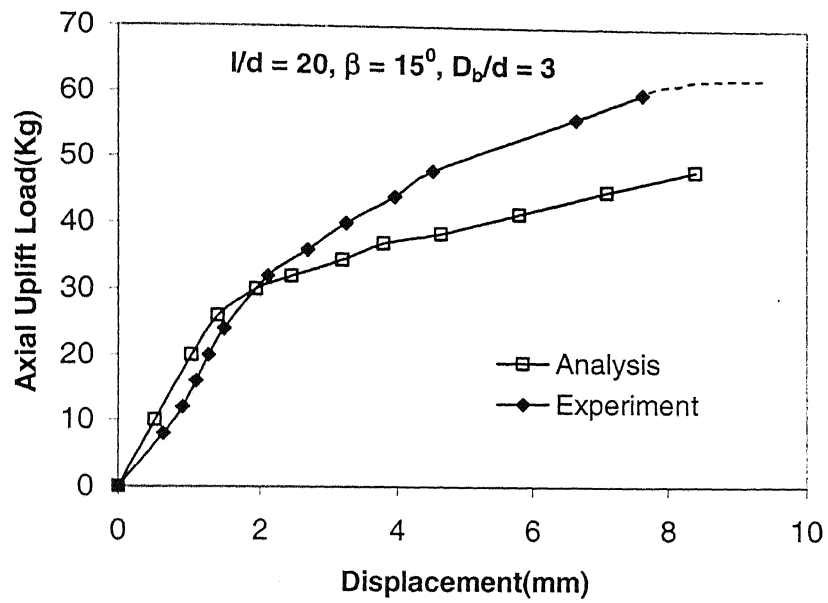


Fig 5.18 Comparison of Load Displacement Curve
($L/d = 20$, $\beta = 15^\circ$, $D_b/d = 3$)

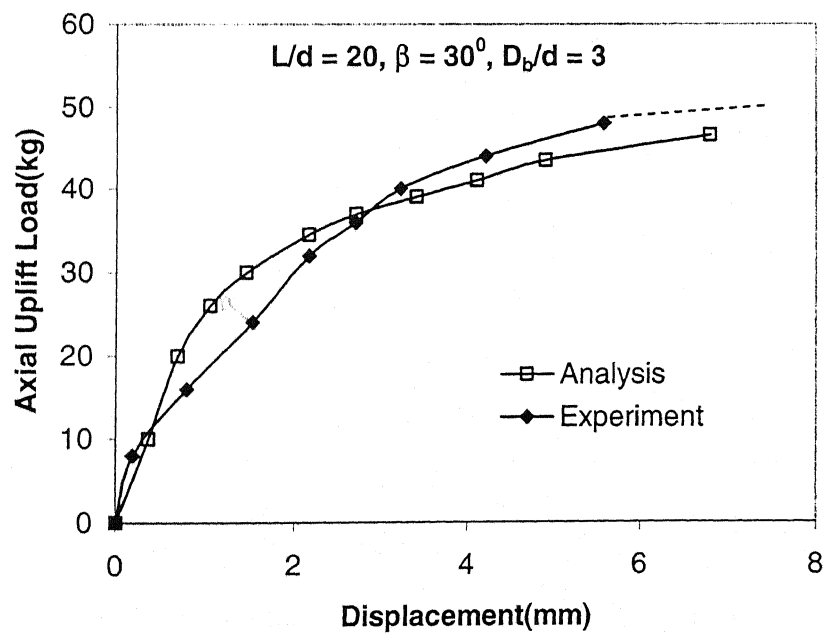


Fig 5.19 Comparison of Load Displacement Curve
($L/d = 20$, $\beta = 30^\circ$, $D_b/d = 3$)

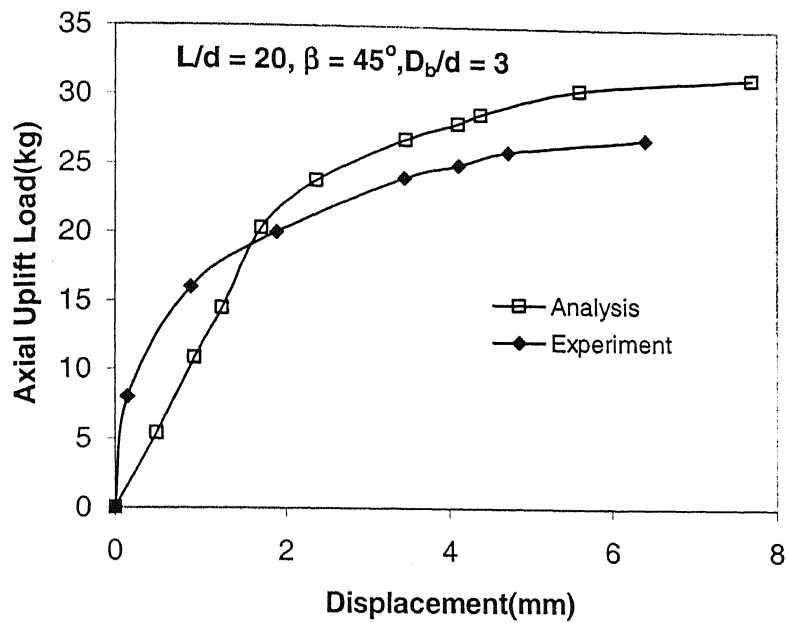


Fig 5.20 Comparison of Load Displacement Curve
($L/d = 20$, $\beta = 45^\circ$, $D_b/d = 3$)

From the above comparison curves it can be observed that at lower load levels, the experimental and analytical load displacement curves are quite similar, and the difference is within the scope of experimental errors and errors due to idealization of real pile behaviour and soil properties. In the linear portion of the load displacement characteristic curves, the analytical solutions give better linearity than the experimental values.

5.3.3 Comparison of Measured and Computed Ultimate Uplift Capacity

In the experimental programs, the ultimate uplift capacity of the pile is taken as the load at which the pile is pulled out. These uplift capacity of inclined anchor piles are compared with those obtained from the modified analysis considering the pile soil slip. In the analysis the ultimate uplift capacity of pile is taken at which all the pile elements are slipped, i.e. the shear stresses along all the pile elements reach the

limiting value. From Fig 5.21 to Fig 5.23, the ultimate uplift capacity of inclined anchor piles are plotted against the batter angles for three different anchor diameters and two different L/d ratios. Both experimental and computed data are given for comparison purpose. As all the experiments are done in dense sand medium (at relative density, $D_R = 76.6\%$, corresponding $\phi = 39^\circ$) and for smooth piles ($\delta = 28^\circ$), the analytical results presents herein are all for dense sand and smooth piles.

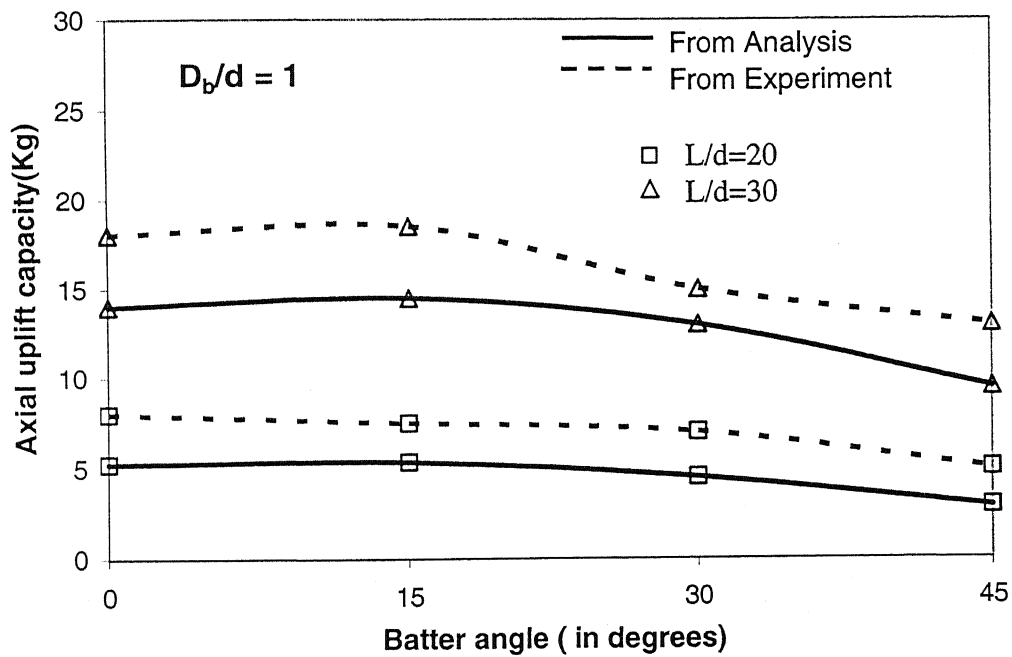


Fig 5.21 Variation of Uplift Capacity with Batter Angle for Pile without Anchor

From the above figures it can be concluded that with increasing the inclination of the pile with the vertical, the ultimate uplift capacity of inclined anchor piles embedded in dense sand medium remains more or less constant up to batter angle 30^0 and then decreases considerably. From both experiments and analyses, the trend obtained is quite similar. The percentage (%) variations of ultimate uplift capacity values computed from present analysis with the measured values from experiments are shown in table 5.1.

Table 5.1 Percentage (%) Difference of Analytical Ultimate Uplift Capacity with Experimental Results Obtained

		Ultimate Uplift capacity		
	Batter Angle (In Degrees)	Analytical	Experimental	% Difference
L/d = 20				
$D_b/d = 1$	15	5.35	7.5	28.67
	30	4.5	7	35.71
	45	2.9	5	42.00
$D_b/d = 2$	15	17.4	22.5	22.67
	30	16	22	27.27
	45	12.5	18	30.56
$D_b/d = 3$	15	48.2	60	19.67
	30	46.5	52	10.58
	45	39.5	42	5.95
L/d = 30				
$D_b/d = 1$	15	14.5	18.5	21.62
	30	13	15	13.33
	45	9.5	13	26.92
$D_b/d = 2$	15	32.5	44.5	26.97
	30	32	40	20.00
	45	24.5	32	23.44
$D_b/d=3$	15	61.75	72	14.24
	30	60.5	66	8.33
	45	52	60	13.33

From Table 5.1 it can be observed that the ultimate uplift capacity values computed by using the proposed elastic-plastic approach lies in the range 5–42% and in most cases the range being 10-30 %. This may be attributed to the assumptions involved in the present analysis, considering the elastic behaviour of the soil and pile material. But the real soils are not ideal elastic materials in that stress and strain are not linearly related, strains are not fully recoverable on reduction of stress, and strains are not independent of time. Another reason is that the experimental ultimate uplift capacity of the pile is taken as the load at which the pile is pulled out according to Pise and Chattopadhyay (1986). But if the ultimate capacity would be assumed to be mobilized when the load displacement curve becomes asymptotic to the displacement axis (Sharma and Pise, 1994) then the ultimate values become at least 10% less than the values obtained here. This order of error in the predicted and measured values may be considered to be within the permissible range.

In any case, as the analytical solutions give ultimate uplift capacity values less than the experimental values, it can be said that the present analysis gives the values on the conservative side.

5.4 Comparative Studies of Uplift Capacity of Inclined Anchor Piles

From the comparison between the experimental results and analytical solutions, it can be concluded that the proposed theory based on elastic approach for obtaining the uplift capacity of inclined anchor piles predicts values which in general follow qualitatively the experimental trend. The difference obtained is only due to the experimental errors and also due to the assumptions taken into account in analysis for idealisation of pile and soil properties. Moreover, the analytical values are on the

conservative side than the experimental values. So the proposed theory can be used to predict the ultimate uplift capacity of the inclined anchor piles correctly.

In the above sections all the analytical solutions presented are for the dense sand and for smooth piles only for comparison purpose. Now the proposed method can be extended to predict the values of uplift capacity of inclined anchor piles embedded in loose and dense sand and also for rough piles. In the following sections, a parametric study on the ultimate pullout capacity of inclined anchor piles are provided based on the analytical solutions obtained for different pile inclinations, different anchor diameters, varying soil density and for rough and smooth piles.

5.4.1 Variation of Uplift Capacity with Batter Angle

In this section the variation of uplift capacity with the batter angle of inclined anchor piles are shown. The variations are shown from Fig 5.24 to Fig 5.29 for three different soil densities, i.e. for dense ($D_R = 78\%$, $\phi = 39^\circ$), medium dense ($D_R = 65\%$, $\phi = 35^\circ$) and loose ($D_R = 35\%$, $\phi = 30^\circ$) conditions. Two anchor diameters are chosen such that D_b/d ratios are 2 and 3 respectively. The plots are given for three L/d ratios 20, 30 and 40.

From Fig 5.24 to Fig 5.29, the following conclusions can be drawn:

- a) For dense sand the uplift capacity values of inclined anchor piles remain almost constant up to 30° batter angle and then decrease considerably for all the L/d ratios of the piles.
- b) For medium dense sand the uplift capacity values decrease gradually with increasing inclination of piles with the vertical.

- c) For loose sand also, the uplift capacity values decrease gradually with increasing inclination of piles with the vertical. However, the rate of decrease in uplift capacity is more in case of loose sand than the rate in medium dense sand.

The same trends are observed for both the anchor diameters.

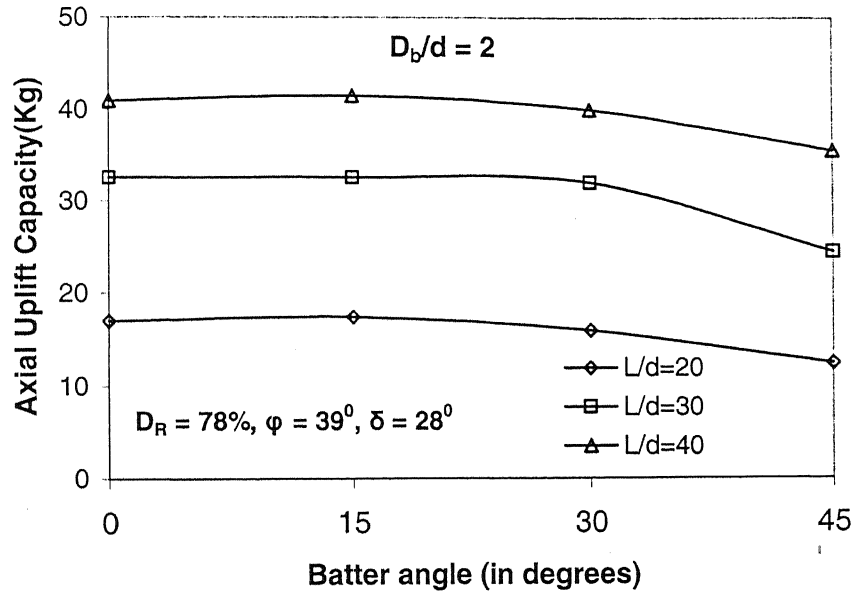


Fig 5.24 Variation of Uplift Capacity with Batter Angle for Dense Sand

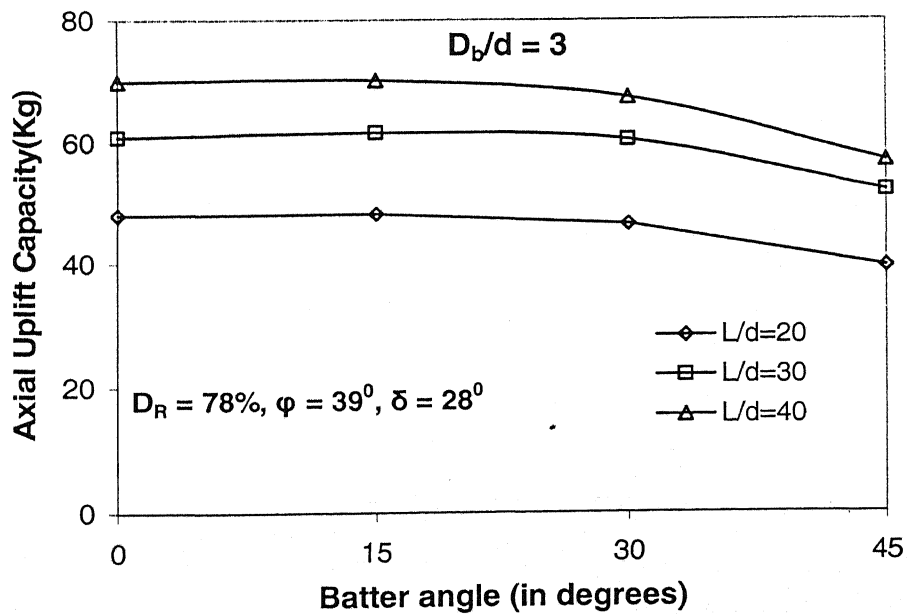


Fig 5.25 Variation of Uplift Capacity with Batter Angle for Dense Sand

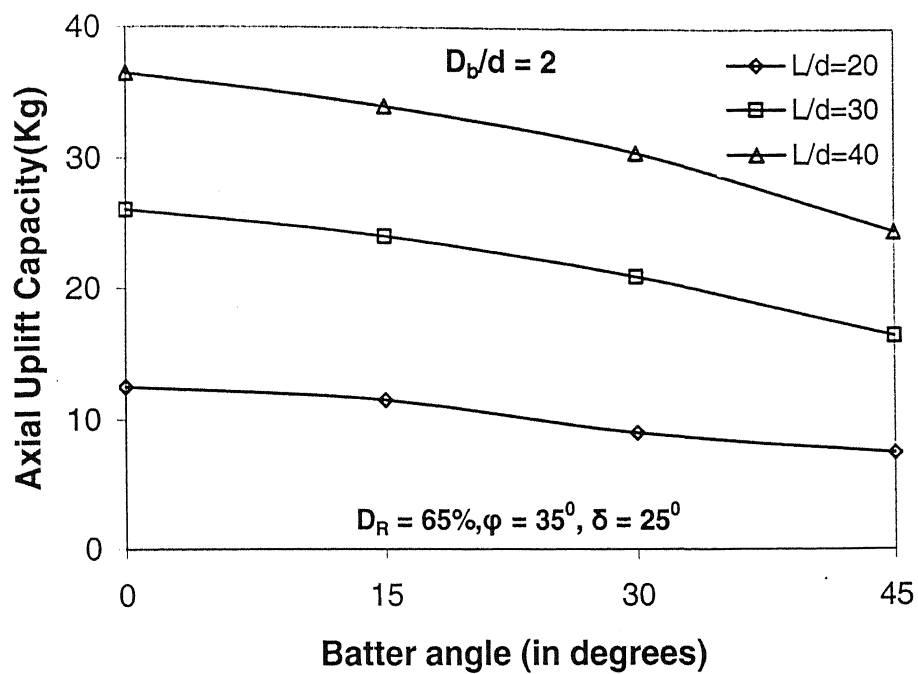


Fig 5.26 Variation of Uplift Capacity with Batter Angle for Medium Dense Sand

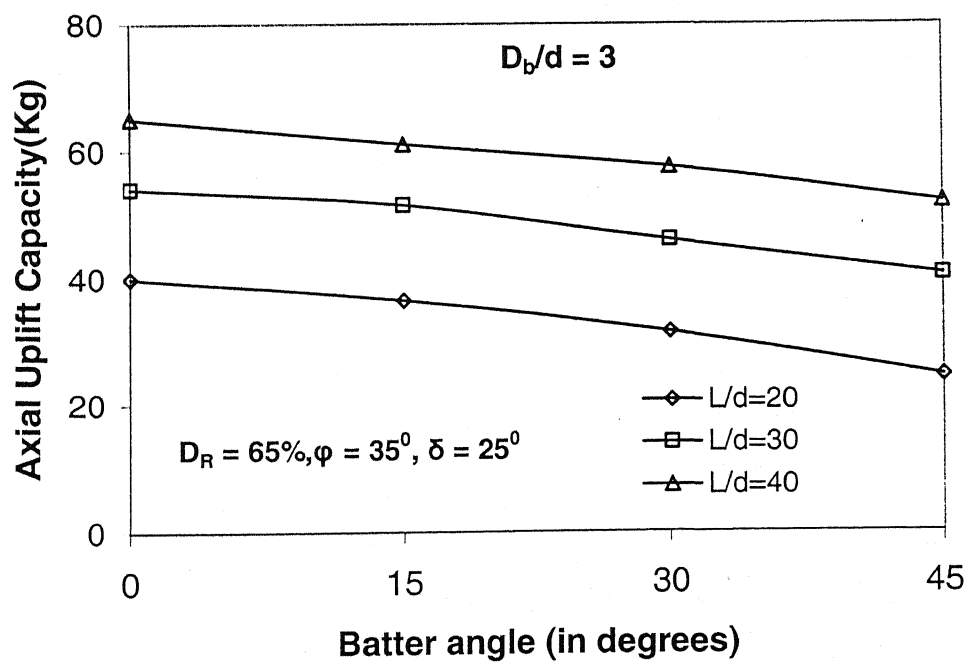


Fig 5.27 Variation of Uplift Capacity with Batter Angle for Medium Dense Sand

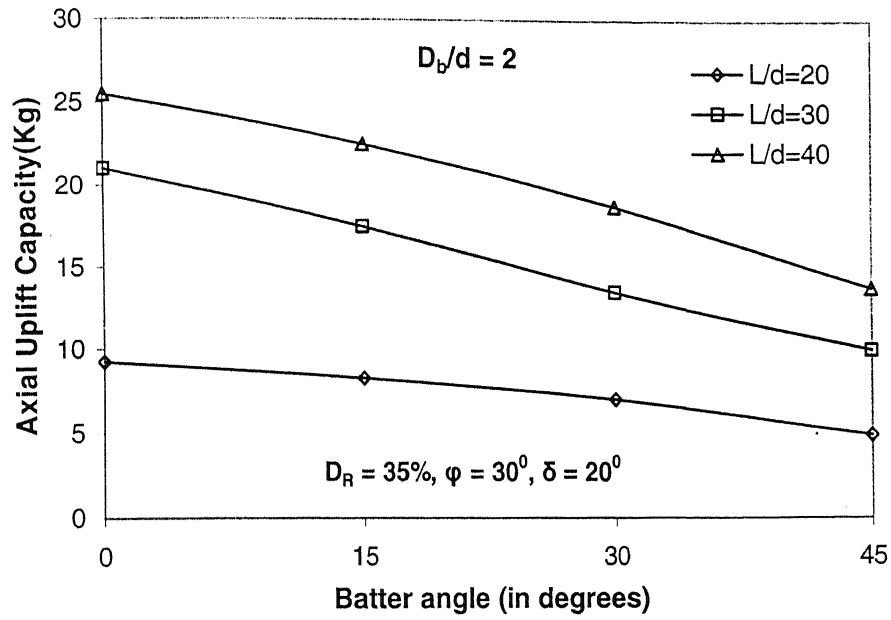


Fig 5.28 Variation of Uplift Capacity with Batter Angle for Loose Sand

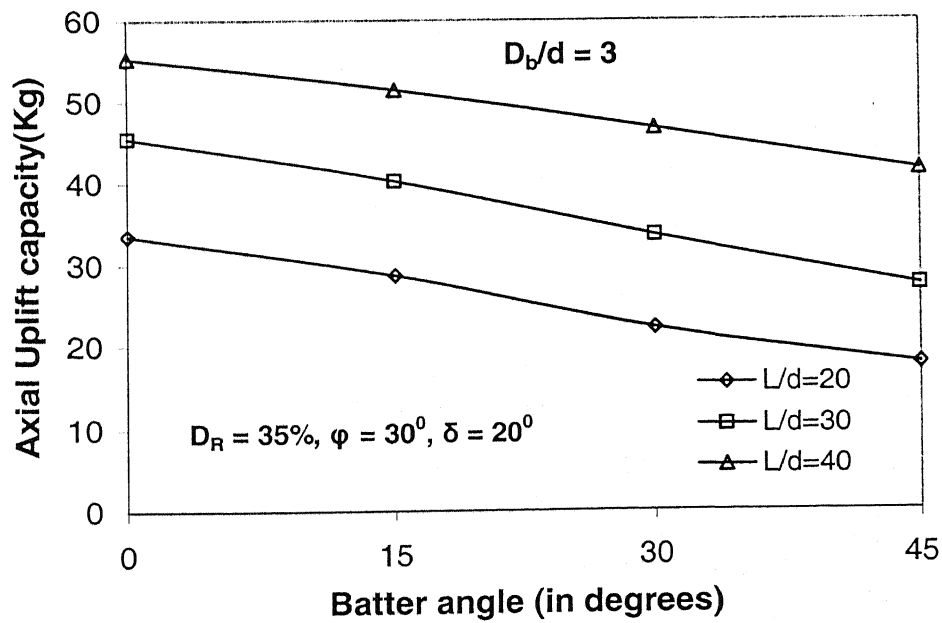


Fig 5.29 Variation of Uplift Capacity with Batter Angle for Loose Sand

5.4.2 Comparison for Smooth and Rough Piles

Till now, the results are shown for smooth piles. The variation of uplift capacities with the batter angle for both rough and smooth piles are shown from Fig 5.30 to Fig 5.32 for dense, medium dense and loose sand medium. The trend obtained for the variation of the uplift capacities with the batter angles is almost identical for both the rough and smooth piles. For dense sand medium, the uplift capacity values of rough piles are almost constant up to 30° angles of batter and then decrease considerably, as obtained for smooth piles also. For medium dense and loose soil medium, the uplift capacity values decreases gradually with increasing the batter angle for both rough and smooth piles. From that it can be said that the pile soil friction angle has no effect in the variation of uplift capacities with the pile inclination, although the ultimate uplift capacity values increase considerably for the rough piles.

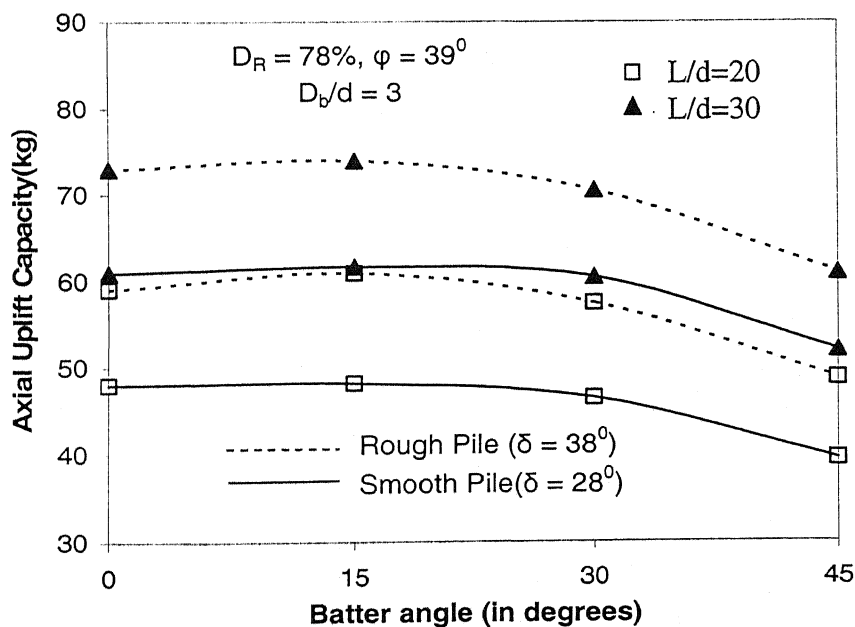


Fig 5.30 Variation of Uplift Capacity with Batter Angle for Comparison between Smooth and Rough pile (in Dense Sand)

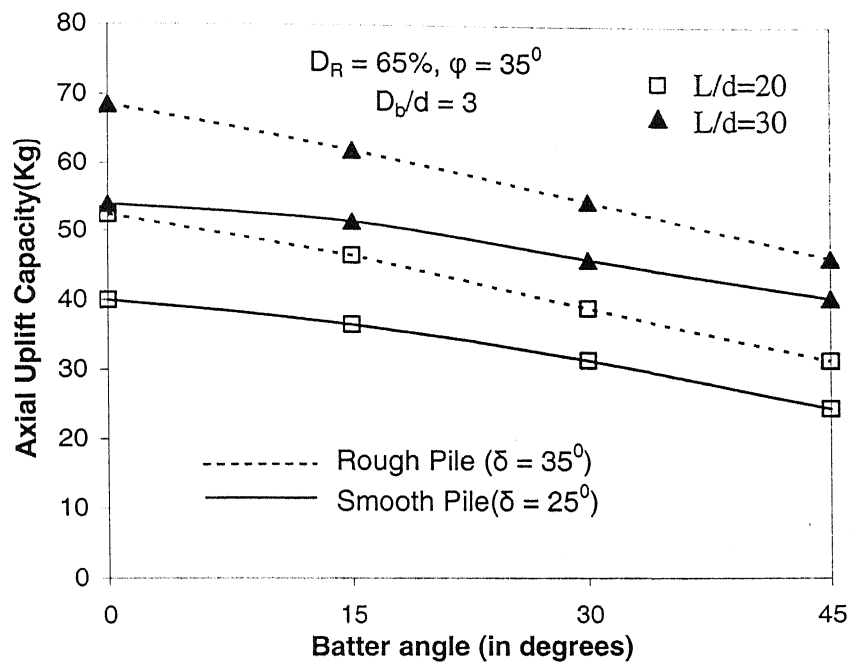


Fig 5.31 Variation of Uplift Capacity with Batter Angle for Comparison between Smooth and Rough pile (in Medium Dense Sand)

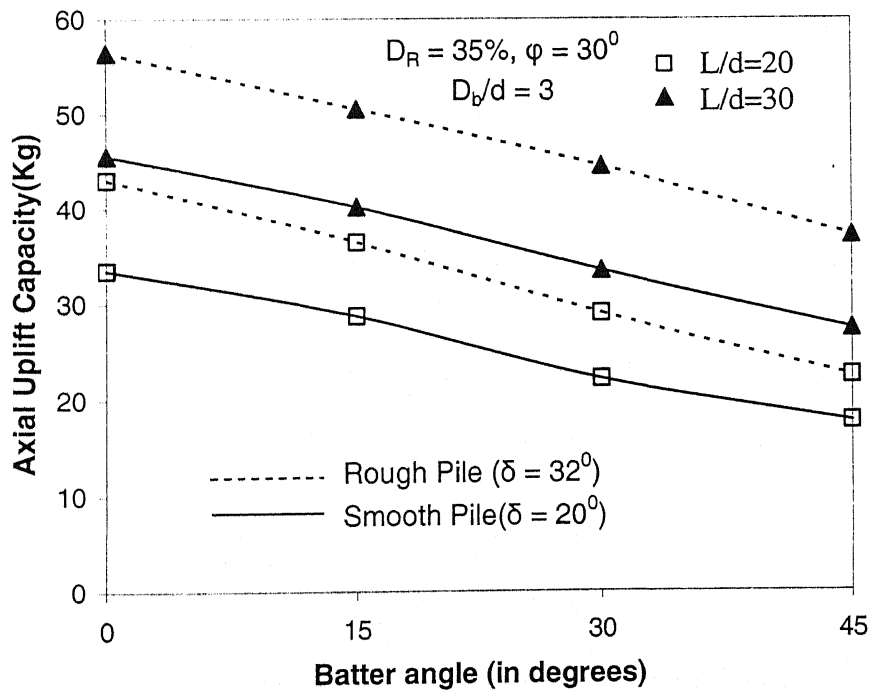


Fig 5.32 Variation of Uplift Capacity with Batter Angle for Comparison between Smooth and Rough pile (in Loose Sand)

Percentage increases in uplift capacity from smooth piles to rough piles versus length to diameter ratio (L/d) are plotted in Fig 5.33 for 15° and 30° inclination of pile respectively. The plots are made for two different anchor diameters such that the ratios D_b/d are 2 and 3. It can be observed that the increase in capacity is higher when $D_b/d = 2$, than when $D_b/d = 3$. That means the increase in uplift capacity with increase in pile friction angle, δ is less for piles having higher D_b/d ratio, i.e. influence of δ decreases as D_b/d ratio increases.

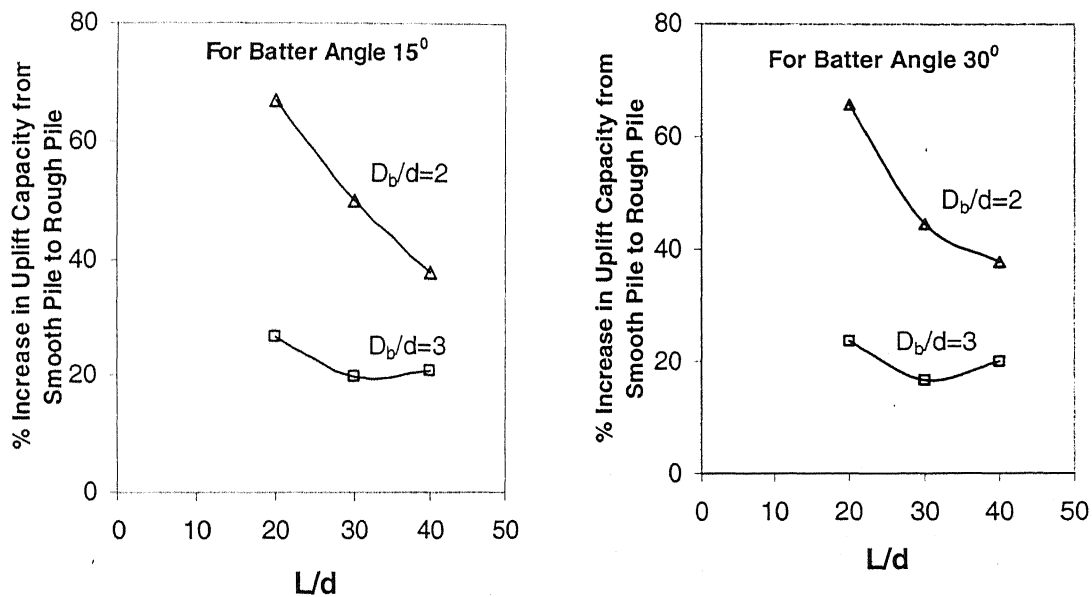


Fig 5.33 Percentage Increase in Capacity from Smooth Piles to Rough Piles

5.4.3 Variation of Uplift Capacity with Anchor Diameter

From Figure 5.34 to Fig 5.42, the variations of ultimate uplift capacity values are plotted against the L/d ratio of the pile for different anchor diameters. D_b/d ratios considered for comparison are 1, 2 and 3 respectively. All the results shown here are for smooth piles. The plots are done for dense, medium dense and loose soil medium. Linear variation is obtained for uplift capacity values with increasing L/d ratio for all anchor diameters.

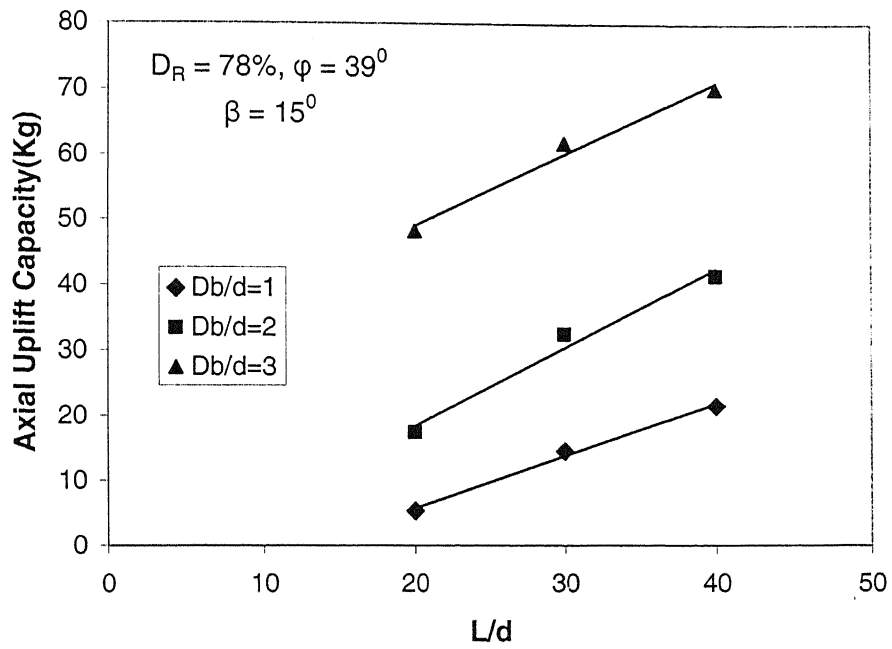


Fig 5.34 Variation of Uplift Capacity with L/d for Dense Sand (For Batter Angle 15°)

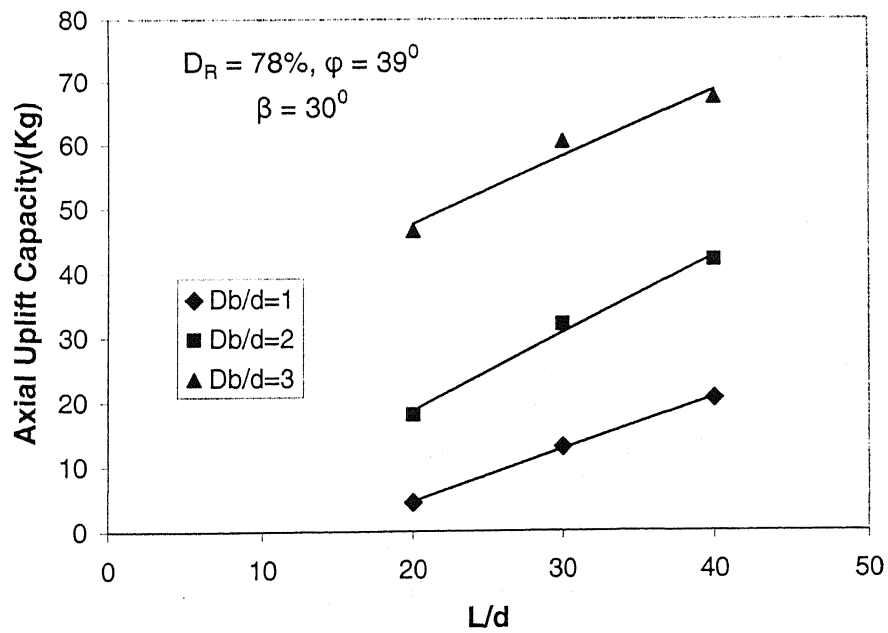


Fig 5.35 Variation of Uplift Capacity with L/d for Dense Sand (For Batter Angle 30°)

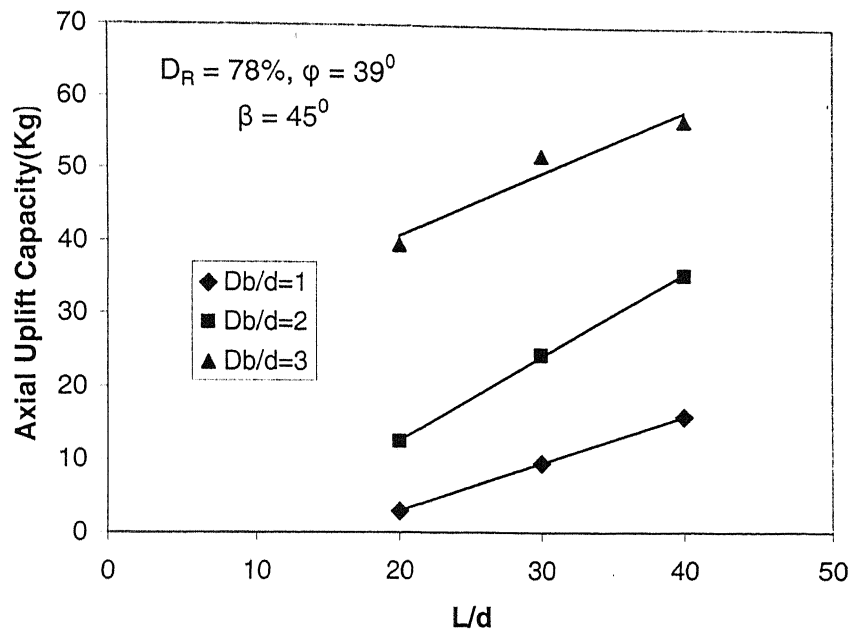


Fig 5.36 Variation of Uplift Capacity with L/d for Dense Sand (For Batter Angle 45°)

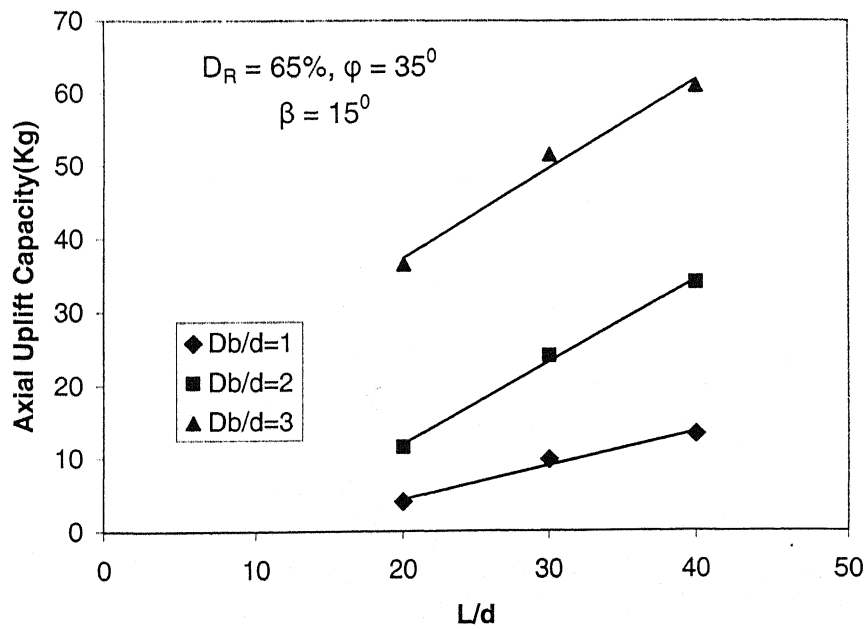


Fig 5.37 Variation of Uplift Capacity with L/d for Medium Dense Sand (For Batter Angle 15°)

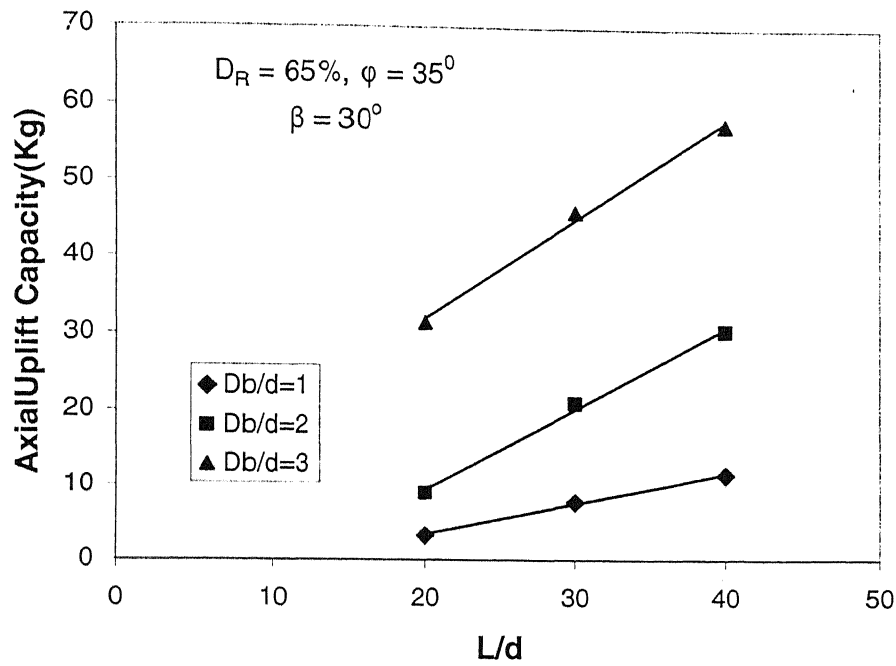


Fig 5.38 Variation of Uplift Capacity with L/d for Medium Dense Sand (For Batter Angle 30°)

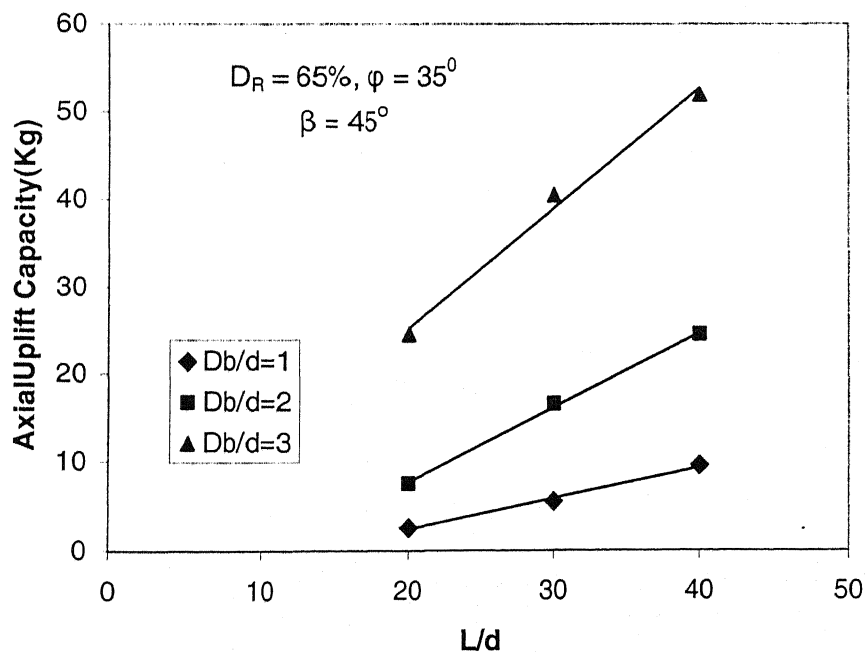


Fig 5.39 Variation of Uplift Capacity with L/d for Medium Dense Sand (For Batter Angle 45°)

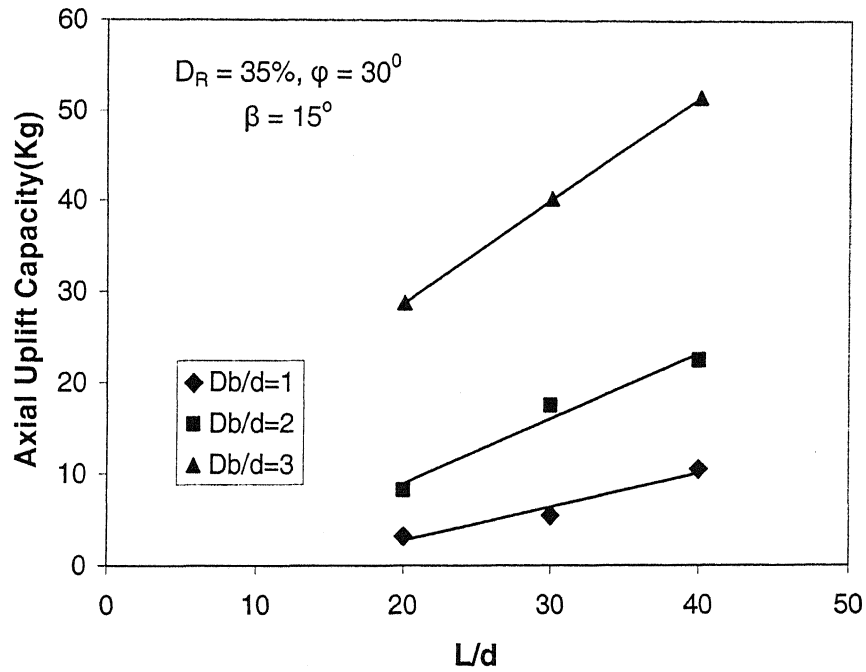


Fig 5.40 Variation of Uplift Capacity with L/d for Loose Sand (For Batter Angle 15°)

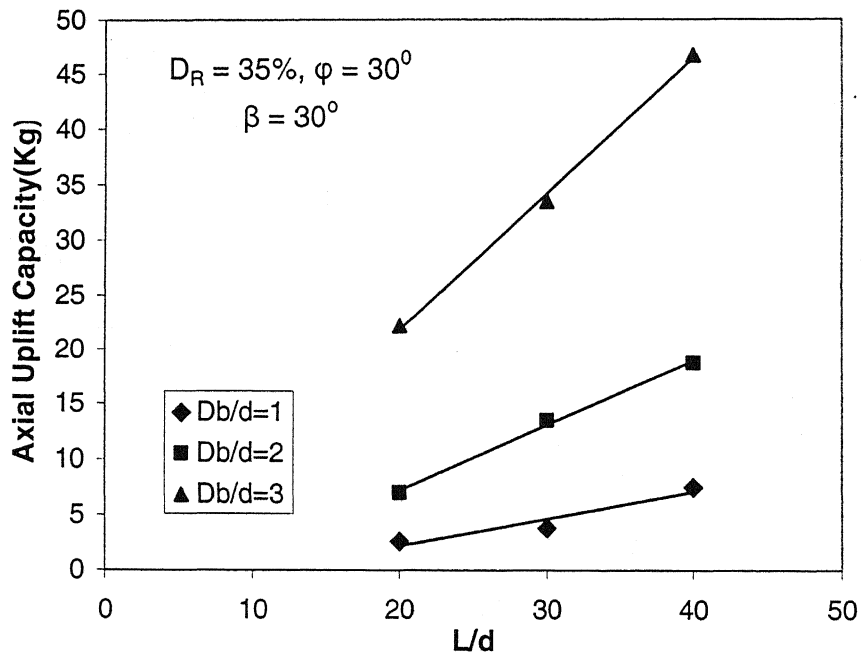


Fig 5.41 Variation of Uplift Capacity with L/d for Loose Sand (For Batter Angle 30°)

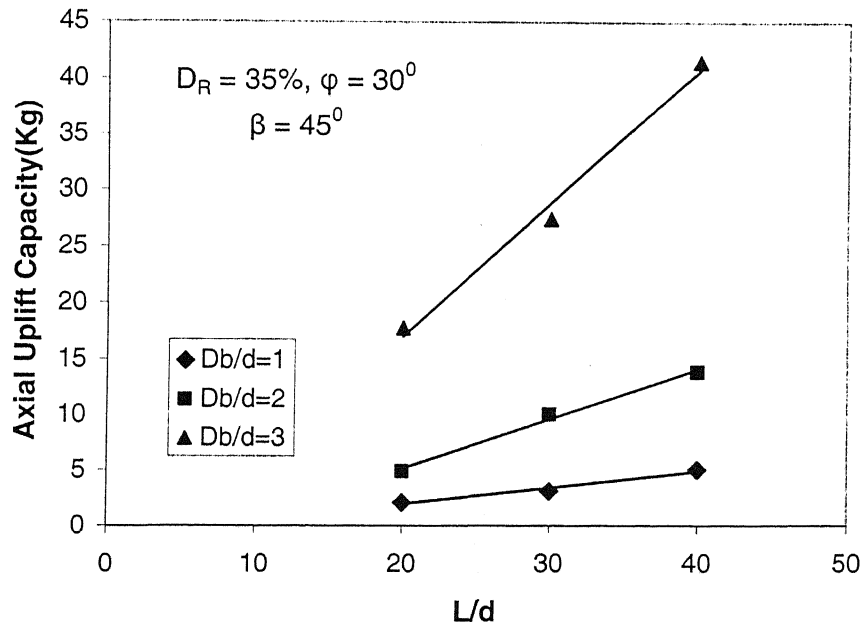


Fig 5.42 Variation of Uplift Capacity with L/d for Loose Sand (For Batter Angle 45°)

5.4.4 Increase in Uplift Capacity Values by Using the Anchors

Figs 5.43 to Fig 5.45 present the variation of uplift capacity ratio (uplift capacity of inclined anchor piles/ uplift capacity of inclined piles without anchor) with L/d ratios for both smooth and rough piles. Figs 5.46 to Fig 5.48 present the variation of uplift capacity ratio (uplift capacity of anchor pile with $D_b/d = 3$ / uplift capacity of anchor piles with $D_b/d = 2$) with L/d ratio. The curves are plotted for three different batter angles of pile. From these figures, it can be observed that the uplift capacity ratio gradually falls down with increase in L/d ratio. In all the cases, the maximum ratio is obtained at L/d ratio equal to 20. From all these figures it can be also observed that the ratio is always higher for a smooth pile than the corresponding rough pile. That is the increase in capacity due to increase in D_b/d ratio is less for rough piles than smoother piles.

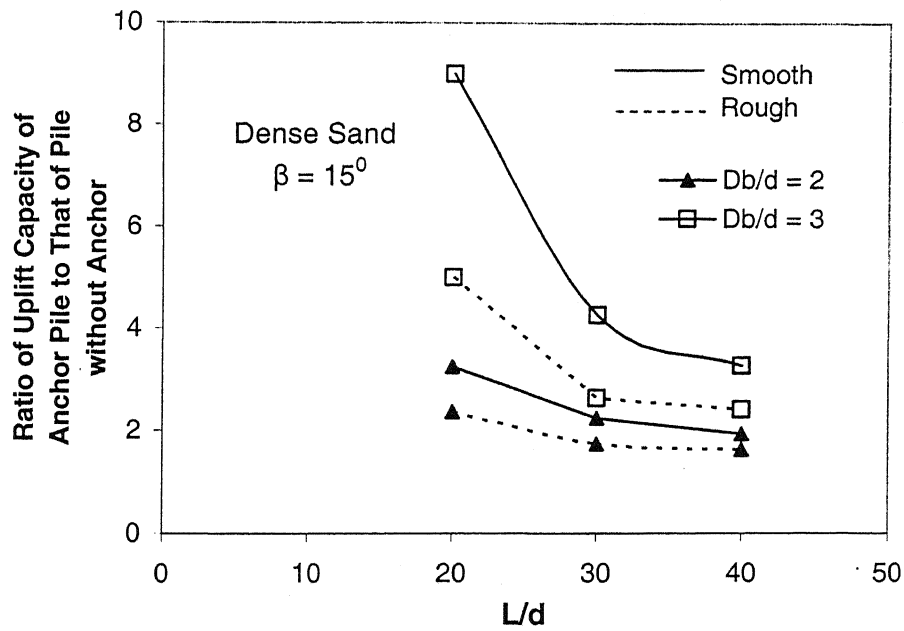


Fig 5.43 Variations of the Ratio of Uplift Capacity of Inclined Anchor Piles to That of Inclined Piles without Anchor (For 15° inclinations)

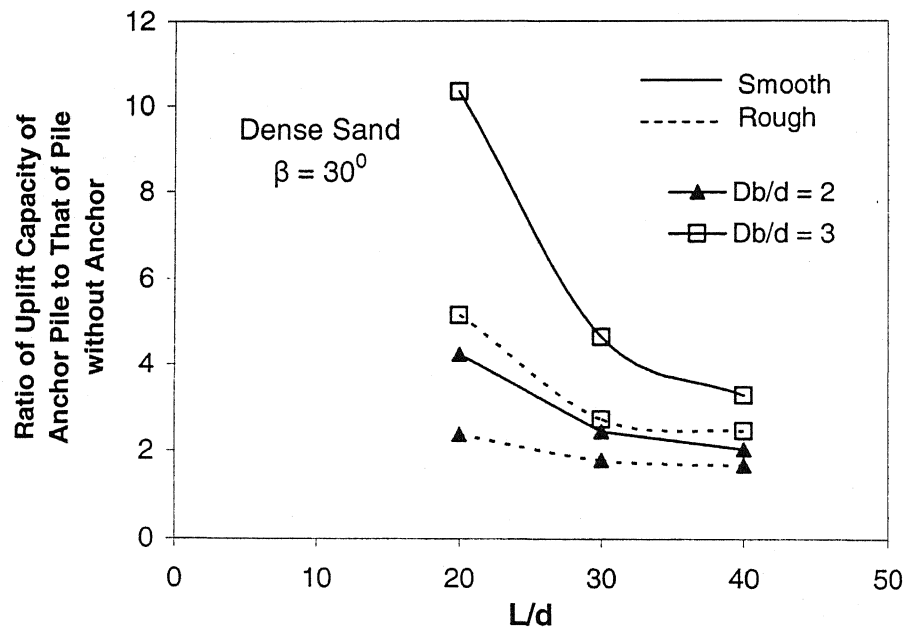


Fig 5.44 Variations of the Ratio of Uplift Capacity of Inclined Anchor Piles to That of Inclined Piles without Anchor (For 30° inclinations)

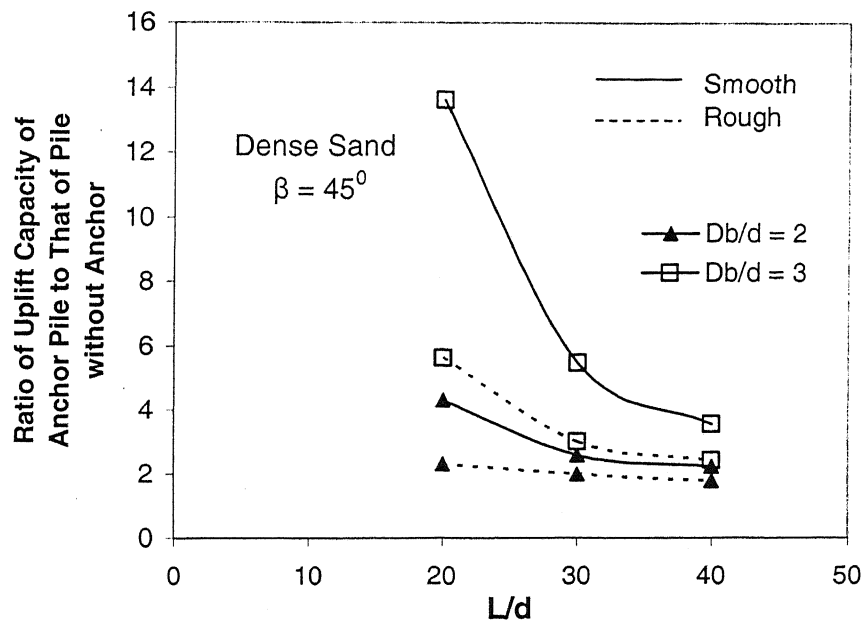


Fig 5.45 Variations of the Ratio of Uplift Capacity of Inclined Anchor Piles to That of Inclined Piles without Anchor (For 45° inclinations)

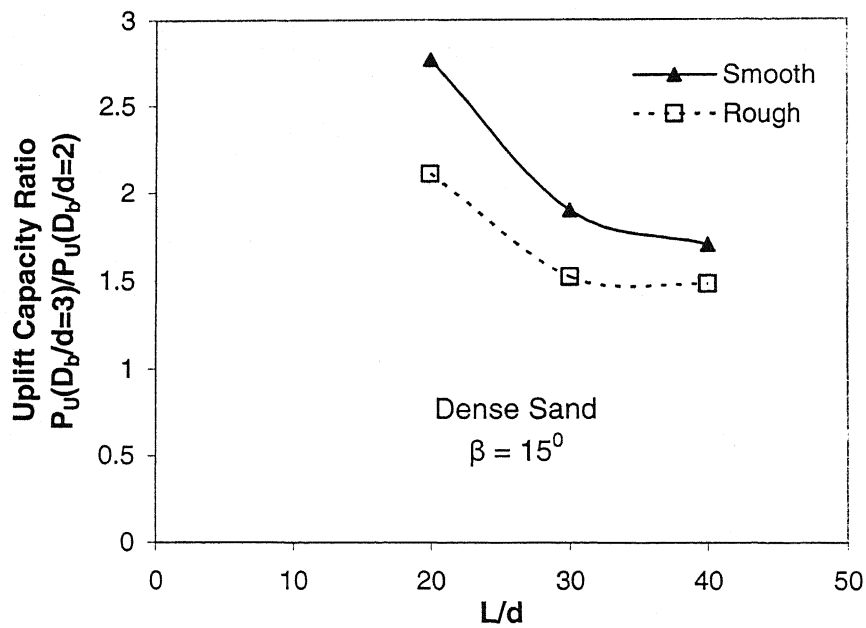


Fig 5.46 Variation of Uplift Capacity Ratio $P_u(D_b/d = 3)/P_u(D_b/d = 2)$ (For 15° inclinations)

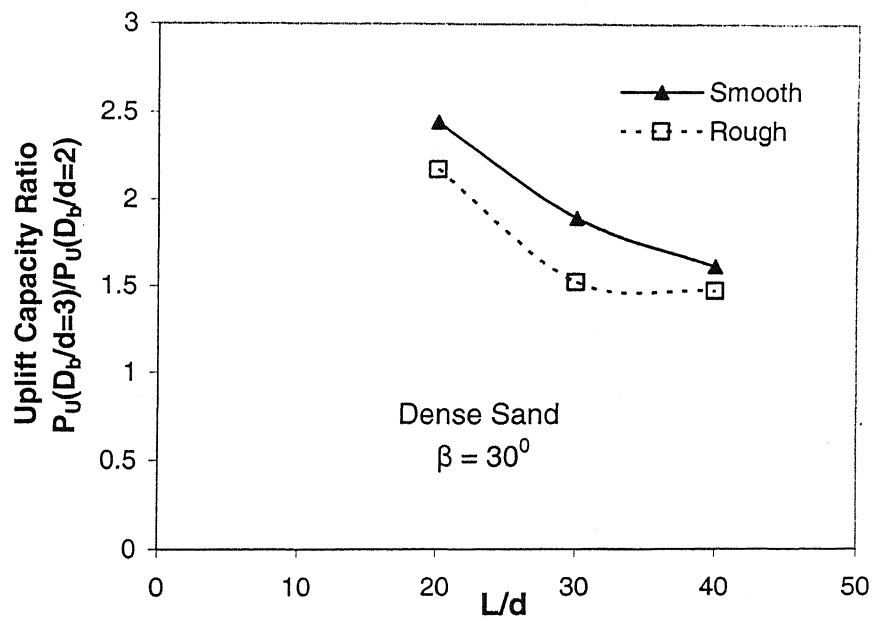


Fig 5.47 Variation of Uplift Capacity Ratio $P_U(D_b/d = 3)/P_U(D_b/d = 2)$
(For 30° inclinations)

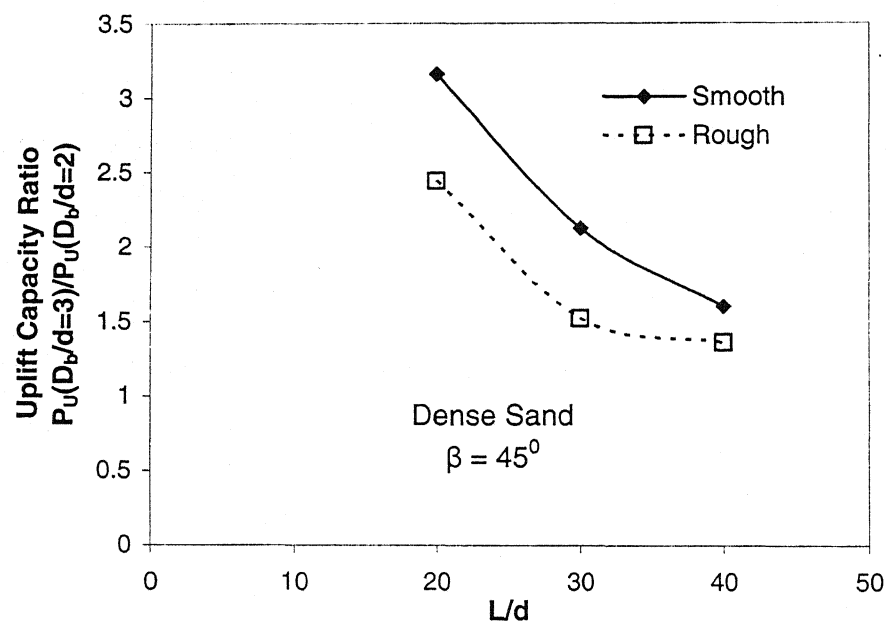


Fig 5.48 Variation of Uplift Capacity Ratio $P_U(D_b/d = 3)/P_U(D_b/d = 2)$
(For 45° inclinations)

CHAPTER 6

CONCLUSIONS AND SCOPE FOR FUTURE STUDY

6.1 General

An analytical method based on the elastic approach has been proposed here to determine the uplift capacity of inclined anchor piles. Based on the analytical solutions and the present experimental results, the following concluding remarks can be drawn.

6.2 Conclusions

- The results obtained from the present approach based on the elastic theory and later modified for pile soil slip to determine the ultimate uplift capacity of the inclined anchor piles are in good agreement with the present experimental results on the same. Thus the present method can be used with confidence in applying such problems.
- From the study on the variation of the ultimate uplift capacity of inclined anchor piles with the batter angle of the pile, the following conclusions are drawn:
 - a) For dense sand the uplift capacity values of inclined anchor piles remain almost constant up to 30^0 batter angle and then decrease considerably for all the L/d ratios of the piles.
 - b) For medium dense sand the uplift capacity values decrease gradually with increasing inclination of piles with the vertical.
 - c) For loose sand also, the uplift capacity values decrease gradually with increasing inclination of piles with the vertical. However, the rate of decrease in uplift capacity is more in case of loose sand than the rate in medium dense sand.

The same trends are observed for both the anchor diameters and both smooth and rough piles.

- For rough piles, the values of ultimate uplift capacity of the inclined anchor piles are more than the smoother piles. On the study of the percentage increase in capacity from smoother piles to rough piles, it is observed that the percentage increase in capacity is higher for D_b/d ratio equal to 2, than when D_b/d ratio equal to 3. That means the increase in uplift capacity with increase in pile friction angle, δ is less for piles having higher D_b/d ratio, i.e. influence of δ decreases as D_b/d ratio increases.
- The axial uplift capacity increases linearly with increase in pile length to diameter (L/d) ratio for a particular anchor diameter and for a particular pile inclination.
- From the study on the variation of the uplift capacity ratio, i.e. the ratio of the uplift capacity of the anchor piles to that of the piles without anchor ($D_b/d = 1$), it is concluded that with increasing L/d ratio, the uplift capacity ratio gradually falls down for both D_b/d equal to 2 and 3. The same trend is obtained for the ratio of the axial uplift capacity of anchor pile with $D_b/d = 3$ to that of anchor piles with $D_b/d = 2$. In all the cases, the maximum ratio is obtained at L/d ratio equal to 20. It is observed that both the uplift capacity ratios are always higher for a smooth pile than that for the rough pile.

6.3 Scope for Future Study

Though it was tried to include most of the variables affecting the uplift behaviour of inclined anchor piles, even there is lots of more considerations to be taken into account in the present analysis. The present analysis can be modified by taking into account those factors. The scopes of the future works are as follows:

- The present analysis assumes constant soil deformation parameters at all the points within the soil. An approximate allowance may be made for the effects of varying soil-deformation moduli (E_s) along the length of the pile.
- The present analysis is only for piles in a semi-infinite soil mass. Approximate analysis can be done for piles in soil layers of finite depth.
- Here the analysis is described only for floating pile. The method can be extended for end-bearing pile to allow for the effect of the stiffer bearing stratum.
- The basic analyses have been formulated in terms of uniform pile with provision for an anchor. However, extensions may be made to allow for cases in which the shaft is not of uniform diameter or in which two or three anchors may be attached to the pile.

APPENDIX

INTEGRATION OF MINDLIN'S EQUATION FOR SOIL DISPLACEMENT ANALYSIS

The soil displacements are evaluated by using the Mindlin's Equation. To determine the elements of $[I_s]$ matrix in Equation (3.4a) Mindlin's Equation is integrated. To carry out this, the pile is divided into n number of elements of equal lengths. In evaluating the soil displacements, the unknown force on each element was resolved into vertical and horizontal components, and vertical and horizontal displacements due to each of these components are calculated using Mindlin's equation. The geometry of the cylindrical pile element is shown in Fig A1. For a general point i in the soil mass, the value of I_{ij} is

$$I_{ij} = 2 \int_{(j-1)\delta}^{j\delta} \int_0^{\pi/2} I_d \theta d\theta dc \dots\dots\dots (A1)$$

Where, ${}_pI$ = influence factor for vertical displacement due to a vertical point load

δ = length of an element = L/n

Referring to Fig A2, the vertical displacement (y) caused by a vertical point load (P_1) within the interior of a semi-infinite elastic isotropic homogeneous mass is given by (Mindlin, 1936),

$$y = \frac{P_1}{16\pi G(1-\nu_s)} \left[\frac{(3-4\nu_s)}{R_1} + \frac{8(1-\nu_s)^2 - (3-4\nu_s)}{R_2} + \frac{(z-c)^2}{R_1^3} \right. \\ \left. + \frac{(3-4\nu_s)(z+c)^2 - 2cz}{R_2^3} + \frac{6cz(z+c)^2}{R_2^5} \right] \dots\dots\dots (A2)$$

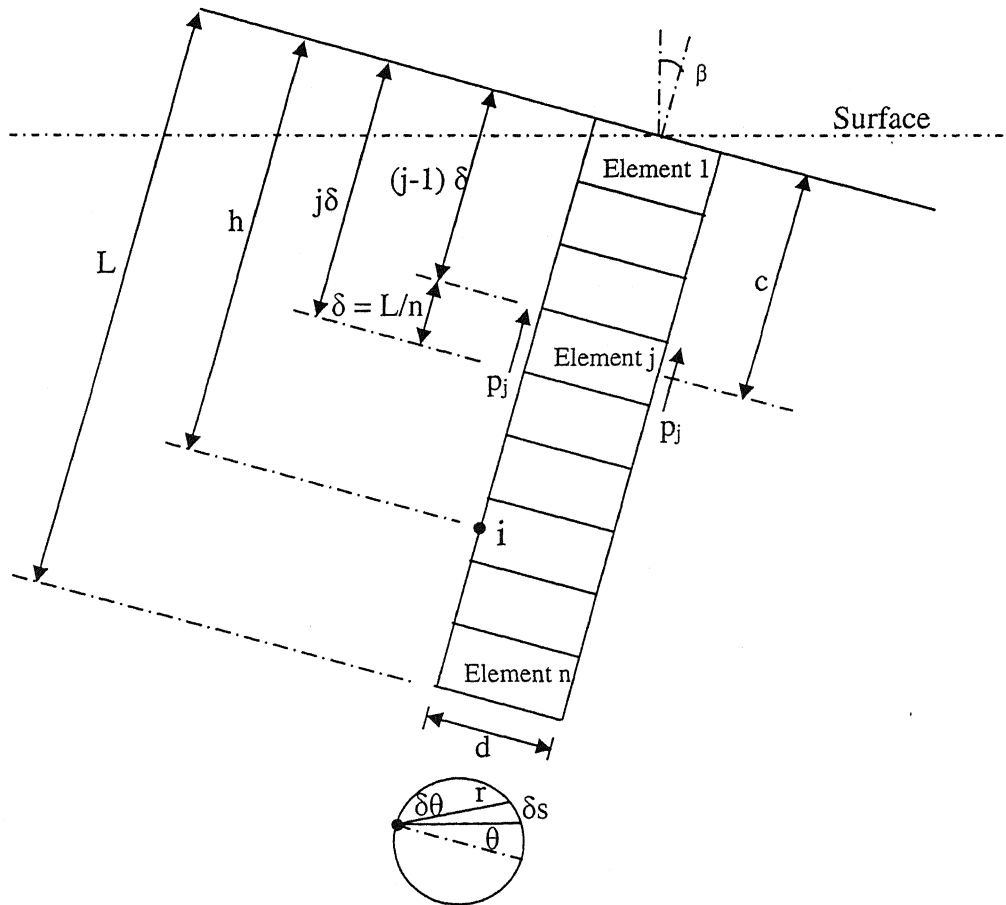


Fig. A1 Single Inclined Pile – Basic Geometry

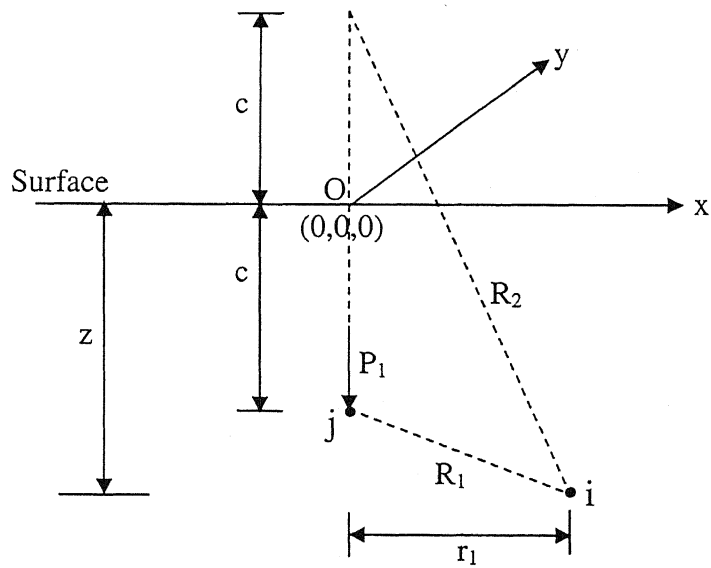


Fig. A2 Definition of Vertical Point Load

From Equation (A2), the influence factor, ${}_pI$ for vertical displacement due to a vertical point load can be derived as

$${}_pI = \frac{(1+\nu_s)}{8\pi(1-\nu_s)} \left\{ \frac{z_1^2}{R_1^3} + \frac{(3-4\nu_s)}{R_1} + \frac{(5-12\nu_s+8\nu_s^2)}{R_2} \right. \\ \left. + \frac{[(3-4\nu_s)z^2 - 2cz + 2c^2]}{R_2^3} + \frac{[6cz^2(z-c)]}{R_2^5} \right\} \dots\dots\dots (A3)$$

Where, referring to Fig A1 for inclined pile,

$$z = (h+c)\cos\beta$$

$$z_1 = (h-c)\cos\beta$$

$$R_1 = \sqrt{(r_1 \cos\beta)^2 + z_1^2}$$

$$R_2 = \sqrt{(r_1 \cos\beta)^2 + z^2}$$

$$r_1 = \sqrt{x^2 + y^2}$$

The integral with respect to c in Equation (A1) can be evaluated analytically to give

$$\int {}_pI dc = \frac{(1+\nu_s)}{8\pi(1-\nu_s)} \left\{ \frac{z_1}{D_1} - 4(1-\nu_s)\ln(z_1+D_1) + 8(1-2\nu_s+\nu_s^2)\ln(z+D) \right. \\ \left. + \frac{[2h^2z/r^2 - 4h - (3-4\nu_s)z]}{D} + \frac{2(hr^2 - h^2z^3/r^2)}{D^3} \right\} \dots\dots\dots (A4)$$

Where, $r = (d \cos\theta)\cos\beta$

$$D_1 = (r^2 + z_1^2)^{1/2}$$

$$D_1 = (r^2 + z^2)^{1/2}$$

And the limits of integration in Equation (A4) are

$$z_1 \text{ from } \{h - (j-1)\delta\}\cos\beta \text{ to } (h - j\delta)\cos\beta$$

$$\text{and } z \text{ from } \{h + (j-1)\delta\}\cos\beta \text{ to } (h + j\delta)\cos\beta$$

The integral with respect to θ is, however, only conveniently evaluated numerically.

The geometry of the pile base is shown in Fig A3. To allow for an enlarged base,, a base radius ($= D_b/2$) different from pile shaft radius is considered.

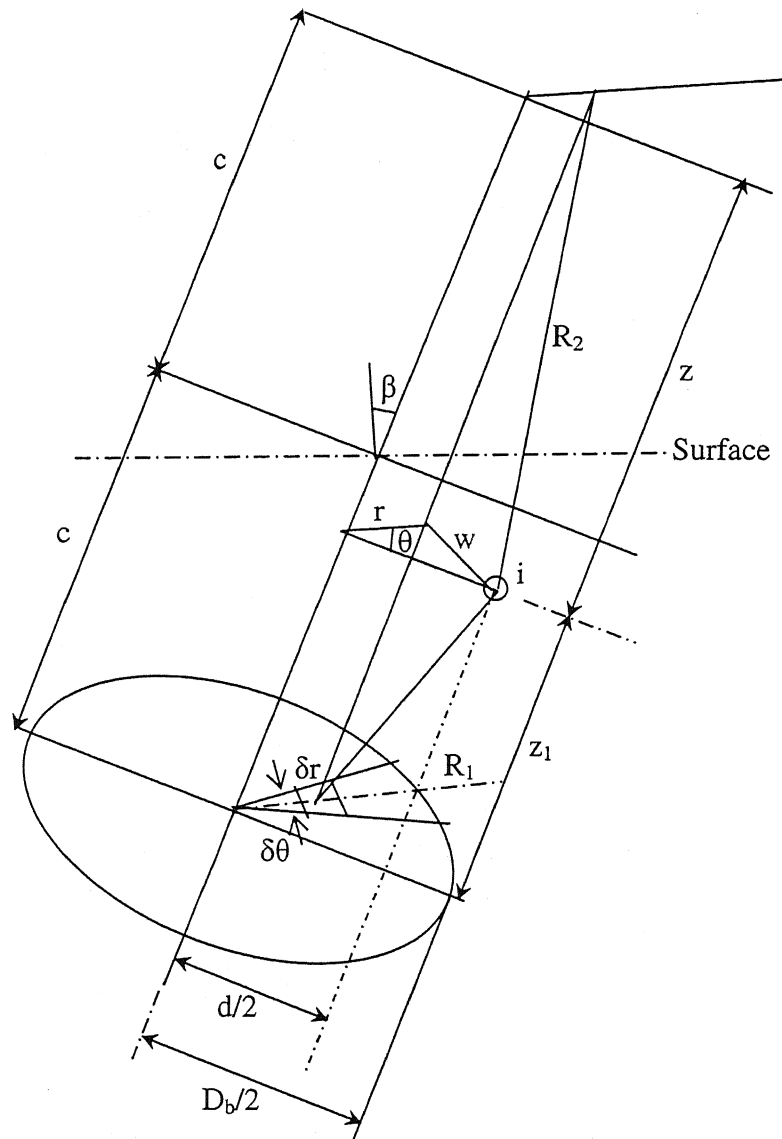


Fig A3 Geometry for Integration over Circular Base

For a general point i in the soil mass,

$$I_{ib} = \frac{1}{d} \int_0^{2\pi D_b/2} \int_0^p I \cdot r dr d\theta \dots\dots\dots (A5)$$

Where pI is given in Equation (A3) and for this case, $c = n\delta = L$.

The integration with respect to r is done analytically and integration with respect to θ is again evaluated by numerical means.

In evaluating the integrals with respect to θ in Equation (A1) and (A5), intervals of $\pi/50$ are usually adequate. To avoid the singularity which occurs when calculating the displacements on the pile shaft for $i = j$, it is most expedient to calculate ordinates at the mid point of each interval and then apply the simple rectangular rule for integration to evaluate the complete double integral. The value of the integral converges to a constant value as the number of intervals in θ is increased, and as mentioned above, intervals of $\pi/50$ are generally sufficient to ensure convergence.

For the center of the base displacement factors resulting from shear stress on the element j ,

$$I_{bj} = \pi \int_{(j-1)\delta}^{j\delta} p I_{dc} \dots\dots\dots (A6)$$

The integral in Equation (A6) is given by Equation (A4), with $h = L$, $D^2 = z^2 + d^2/4$ and $D_1^2 = z_1^2 + d^2/4$.

For the vertical displacement of the base resulting from the base itself, it is desirable to make an approximation allowance for the effect of the rigidity of the base by multiplying the displacement of the center of the uniformly loaded circular base by a factor of $\pi/4$; this is the ratio of the surface displacement of a rigid circle on the surface of a half-space to the center displacement of a corresponding uniformly loaded circle, and may be assumed to apply approximately to embedded areas. Thus,

$$I_{bb} = \frac{\pi}{4} 2 \frac{\pi}{d} \int_{d/2}^{D_b/2} p I_{rd} dr \dots\dots\dots (A7)$$

Where, pI is given in Equation (A4).

For the horizontal displacements caused by the horizontal component of the stresses on the pile shaft, Mindlin's basic equation for the horizontal displacements caused by a horizontal point load within the interior of a semi-infinite elastic, isotropic, homogeneous mass is referred. Referring to Fig A4, this solution is as follows:

$$\rho_x = \frac{Q}{16\pi G(1-\nu_s)} \left[\frac{(3-4\nu_s)}{R_1} + \frac{1}{R_2} + \frac{x^2}{R_1^3} + \frac{(3-4\nu_s)x^2}{R_2^3} + \frac{2cz}{R_2^3} \left(1 - \frac{3x^2}{R_2^2} \right) + \frac{4(1-\nu_s)(1-2\nu_s)}{R_2+z+c} \times \left(1 - \frac{x^2}{R_2(R_2+z+c)} \right) \right] \dots\dots\dots (A8)$$

This equation is followed to determine the horizontal displacement component.

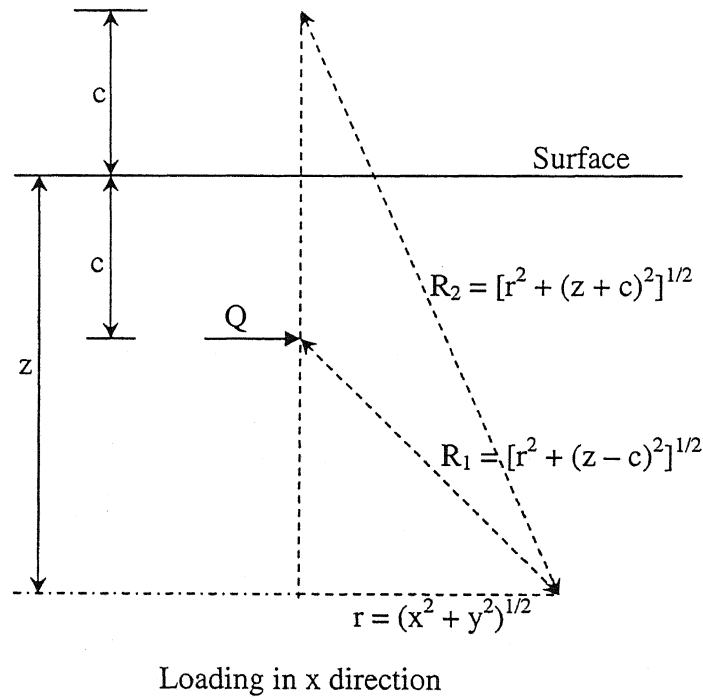


Fig A4 Definition of Horizontal Point Load

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